EXERCISE SET 2.5

1. (a) no, $x = 2$ (b) no, $x = 2$ (c) no, $x = 2$ (d) yes (e) yes (f) yes

2. (a) no, $x = 2$ (b) no, $x = 2$ (c) no, $x = 2$ (d) yes (e) no, $x = 2$ (f) yes

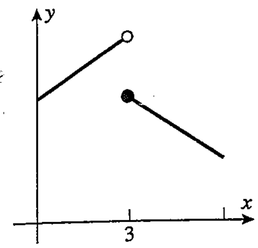
3. (a) no, $x = 1, 3$ (b) yes (c) no, $x = 1$ (d) yes (e) no, $x = 3$ (f) yes

4. (a) no, $x = 3$ (b) yes (c) yes (d) yes
(e) no, $x = 3$ (f) yes

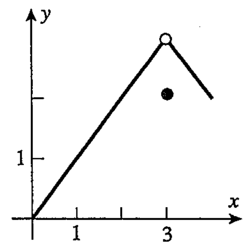
5. (a) 3 (b) 3

6. $-2/5$

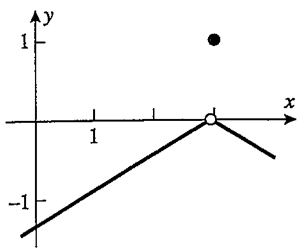
7. (a)



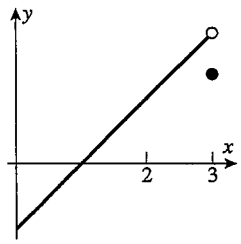
(b)



(c)

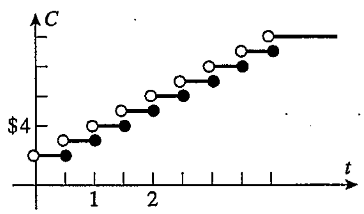


(d)



8. $f(x) = 1/x$, $g(x) = \begin{cases} 0 & \text{if } x = 0 \\ \sin \dfrac{1}{x} & \text{if } x \neq 0 \end{cases}$

9. (a)



(b) One second could cost you one dollar.

10. (a) no; disasters (war, flood, famine, pestilence, for example) can cause discontinuities
(b) continuous
(c) not usually continuous; see Exercise 9
(d) continuous

11. none

12. none

13. none

14. $x = -2, 2$

15. $x = 0, -1/2$

16. none

17. $x = -1, 0, 1$

18. $x = -4, 0$

19. none

20. $x = -1, 0$

21. none; $f(x) = 2x + 3$ is continuous on $x < 4$ and $f(x) = 7 + \dfrac{16}{x}$ is continuous on $4 < x$; $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = f(4) = 11$ so f is continuous at $x = 4$

22. $\lim_{x \to 1} f(x)$ does not exist so f is discontinuous at $x = 1$

23. (a) f is continuous for $x < 1$, and for $x > 1$; $\lim_{x \to 1^-} f(x) = 5$, $\lim_{x \to 1^+} f(x) = k$, so if $k = 5$ then f is continuous for all x
(b) f is continuous for $x < 2$, and for $x > 2$; $\lim_{x \to 2^-} f(x) = 4k$, $\lim_{x \to 2^+} f(x) = 4 + k$, so if $4k = 4 + k$, $k = 4/3$ then f is continuous for all x

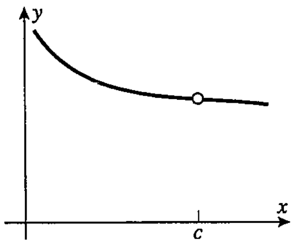
24. (a) f is continuous for $x < -3$, and for $x > -3$; $\lim_{x\to(-3)^-} f(x) = k/9$, $\lim_{x\to(-3)^+} f(x) = 0$, so if $k = 0$ then f is continuous for all x

(b) f is continuous for $x < 0$, and for $x > 0$; $\lim_{x\to 0^-} f(x)$ doesn't exist unless $k = 0$, and if so then $\lim_{x\to 0^-} f(x) = +\infty$; $\lim_{x\to 0^+} f(x) = 9$, so no value of k.

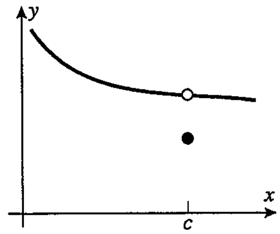
25. f is continuous for $x < -1$, $-1 < x < 2$ and $x > 2$; $\lim_{x\to -1^-} f(x) = 4$, $\lim_{x\to -1^+} f(x) = k$, so $k = 4$ is required. Next, $\lim_{x\to 2^-} f(x) = 3m + k = 3m + 4$, $\lim_{x\to 2^+} f(x) = 9$, so $3m + 4 = 9, m = 5/3$ and f is continuous everywhere if $k = 4, m = 5/3$

26. (a) no, f is not defined at $x = 2$
(b) no, f is not defined for $x \le 2$
(c) yes
(d) no, f is not defined for $x \le 2$

27. (a)

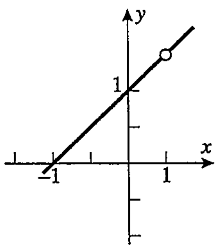


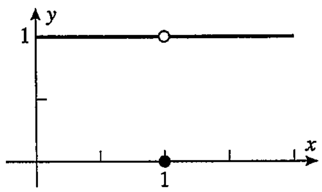
(b)



28. (a) $f(c) = \lim_{x\to c} f(x)$

(b) $\lim_{x\to 1} f(x) = 2$ $\lim_{x\to 1} g(x) = 1$





(c) Define $f(1) = 2$ and redefine $g(1) = 1$.

29. (a) $x = 0$, $\lim_{x\to 0^-} f(x) = -1 \ne +1 = \lim_{x\to 0^+} f(x)$ so the discontinuity is not removable

(b) $x = -3$; define $f(-3) = -3 = \lim_{x\to -3} f(x)$, then the discontinuity is removable

(c) f is undefined at $x = \pm 2$; at $x = 2$, $\lim_{x\to 2} f(x) = 1$, so define $f(2) = 1$ and f becomes continuous there; at $x = -2$, $\lim_{x\to -2}$ does not exist, so the discontinuity is not removable

30. (a) f is not defined at $x = 2$; $\lim_{x\to 2} f(x) = \lim_{x\to 2} \dfrac{x+2}{x^2+2x+4} = \dfrac{1}{3}$, so define $f(2) = \dfrac{1}{3}$ and f becomes continuous there

(b) $\lim_{x\to 2^-} f(x) = 1 \ne 4 = \lim_{x\to 2^+} f(x)$, so f has a nonremovable discontinuity at $x = 2$

(c) $\lim_{x\to 1} f(x) = 8 \ne f(1)$, so f has a removable discontinuity at $x = 1$