EXERCISE SET 2.5

1. (a) no, $x = 2$
   (e) yes
   (b) no, $x = 2$
   (f) yes
   (c) no, $x = 2$
   (d) yes
2. (a) no, $x = 2$
   (e) no, $x = 2$
   (b) no, $x = 2$
   (f) yes
   (c) no, $x = 2$
   (d) yes
3. (a) no, $x = 1, 3$
   (e) no, $x = 3$
   (b) yes
   (f) yes
   (c) no, $x = 1$
   (d) yes
8. \( f(x) = \frac{1}{x}, \quad g(x) = \begin{cases} 
0 & \text{if } x = 0 \\
\sin \frac{1}{x} & \text{if } x \neq 0 
\end{cases} \)

9. (a) \[
\begin{array}{c}
y = \text{?} \\
1 \\
3 \\
\infty \\
\text{x}
\end{array}
\]
(b) One second could cost you one dollar.

10. (a) no; disasters (war, flood, famine, pestilence, for example) can cause discontinuities
(b) continuous
(c) not usually continuous; see Exercise 9
(d) continuous

11. none
12. none
13. none
14. \( x = -2, 2 \)
15. \( x = 0, -1/2 \)
16. none
17. \( x = -1, 0, 1 \)
18. \( x = -4, 0 \)
19. none
20. \( x = -1, 0 \)

21. none; \( f(x) = 2x + 3 \) is continuous on \( x < 4 \) and \( f(x) = 7 + \frac{16}{x} \) is continuous on \( 4 < x \);
\[
\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = f(4) = 11 \text{ so } f \text{ is continuous at } x = 4
\]

22. \( \lim_{x \to 1} f(x) \) does not exist so \( f \) is discontinuous at \( x = 1 \)

23. (a) \( f \) is continuous for \( x < 1 \), and for \( x > 1 \); \( \lim_{x \to 1^-} f(x) = 5 \), \( \lim_{x \to 1^+} f(x) = k \), so if \( k = 5 \) then \( f \) is continuous for all \( x \)
(b) \( f \) is continuous for \( x < 2 \), and for \( x > 2 \); \( \lim_{x \to 2^-} f(x) = 4k \), \( \lim_{x \to 2^+} f(x) = 4 + k \), so if \( 4k = 4 + k \), \( k = 4/3 \) then \( f \) is continuous for all \( x \)
24. (a) \( f \) is continuous for \( x < -3 \), and for \( x > -3 \); \( \lim_{x \to (-3)^-} f(x) = k/9 \), \( \lim_{x \to (-3)^+} f(x) = 0 \), so if \( k = 0 \) then \( f \) is continuous for all \( x \).

(b) \( f \) is continuous for \( x < 0 \), and for \( x > 0 \); \( \lim_{x \to 0^-} f(x) \) doesn’t exist unless \( k = 0 \), and if so then \( \lim_{x \to 0^-} f(x) = +\infty \); \( \lim_{x \to 0^+} f(x) = 9 \), so no value of \( k \).

25. \( f \) is continuous for \( x < -1 \), \( -1 < x < 2 \) and \( x > 2 \); \( \lim_{x \to -1^-} f(x) = 4 \), \( \lim_{x \to -1^+} f(x) = k \), so \( k = 4 \) is required. Next, \( \lim_{x \to 2^-} f(x) = 3m + k = 3m + 4 \), \( \lim_{x \to 2^+} f(x) = 9 \), so \( 3m + 4 = 9 \), \( m = 5/3 \) and \( f \) is continuous everywhere if \( k = 4 \), \( m = 5/3 \).

26. (a) no, \( f \) is not defined at \( x = 2 \)

(c) yes

(b) no, \( f \) is not defined for \( x \leq 2 \)

(d) no, \( f \) is not defined for \( x \leq 2 \)

27. (a) \[ y \]

(b) \[ y \]

28. (a) \( f(c) = \lim_{x \to c} f(x) \)

(b) \( \lim_{x \to 1^-} f(x) = 2 \), \( \lim_{x \to 1^+} g(x) = 1 \)

(c) Define \( f(1) = 2 \) and redefine \( g(1) = 1 \).

29. (a) \( x = 0 \), \( \lim_{x \to 0^-} f(x) = -1 \neq +1 = \lim_{x \to 0^+} f(x) \) so the discontinuity is not removable.

(b) \( x = -3 \); define \( f(-3) = -3 = \lim_{x \to -3} f(x) \), then the discontinuity is removable.

(c) \( f \) is undefined at \( x = \pm 2 \); at \( x = 2 \), \( \lim_{x \to 2^-} f(x) = 1 \), so define \( f(2) = 1 \) and \( f \) becomes continuous there; at \( x = -2 \), \( \lim \) does not exist, so the discontinuity is not removable.

30. (a) \( f \) is not defined at \( x = 2 \); \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2} \frac{x + 2}{x^2 + 2x + 4} = \frac{1}{3} \), so define \( f(2) = \frac{1}{3} \) and \( f \) becomes continuous there.

(b) \( \lim_{x \to 2^-} f(x) = 1 \neq 4 = \lim_{x \to 2^+} f(x) \), so \( f \) has a nonremovable discontinuity at \( x = 2 \).

(c) \( \lim_{x \to 1^-} f(x) = 8 \neq f(1) \), so \( f \) has a removable discontinuity at \( x = 1 \).