EXERCISE SET 1.6

1. (a) -4 (b) 4 (c) $1/4$

2. (a) $1/16$ (b) 8 (c) $1/3$

3. (a) 2.9690 (b) 0.0341

4. (a) 1.8882 (b) 0.9381

5. (a) $\log_2 16 = \log_2(2^4) = 4$ (b) $\log_2\left(\frac{1}{32}\right) = \log_2(2^{-5}) = -5$
   (c) $\log_4 4 = 1$ (d) $\log_9 3 = \log_9(9^{1/2}) = 1/2$

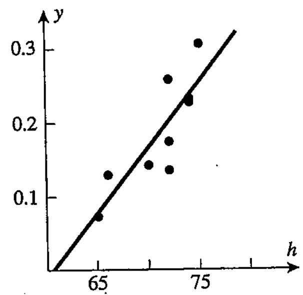
6. (a) $\log_{10}(0.001) = \log_{10}(10^{-3}) = -3$ (b) $\log_{10}(10^4) = 4$
   (c) $\ln(e^3) = 3$ (d) $\ln(\sqrt{e}) = \ln(e^{1/2}) = 1/2$

7. (a) 1.3655 (b) -0.3011

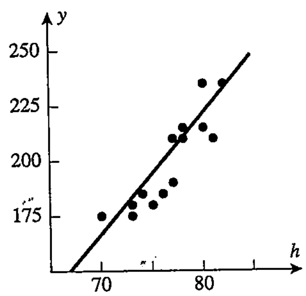
8. (a) -0.5229 (b) 1.1447

9. (a) $2\ln a + \frac{1}{2}\ln b + \frac{1}{2}\ln c = 2r + s/2 + t/2$ (b) $\ln b - 3\ln a - \ln c = s - 3r - t$

10. (a) $\frac{1}{3}\ln c - \ln a - \ln b = t/3 - r - s$ (b) $\frac{1}{2}(\ln a + 3\ln b - 2\ln c) = r/2 + 3s/2 - t$

11. (a) Let h denote the height in inches and y the number of rebounds per minute. Then $y = 0.0174h - 1.0549,\ r = 0.8423$
    (b)
    

    (c) The least squares line is a fair model for these data, since the correlation coefficient is 0.8423.

12. (a) Let h denote the height in inches and y the weight in pounds. Then $y = 5.45h - 218;\ r = 0.8985$
    (b)
    

    (c) 239.73 lb

13. $\log\dfrac{2^4(16)}{3} = \log(256/3)$

14. $\log\sqrt{x} - \log(\sin^3 2x) + \log 100 = \log\dfrac{100\sqrt{x}}{\sin^3 2x}$

15. $\ln\dfrac{\sqrt[3]{x}(x+1)^2}{\cos x}$

16. $1 + x = 10^3 = 1000,\ x = 999$

17. $\sqrt{x} = 10^{-1} = 0.1,\ x = 0.01$

18. $x^2 = e^4,\ x = \pm e^2$

19. $1/x = e^{-2}$, $x = e^2$

20. $x = 7$

21. $2x = 8$, $x = 4$

22. $\log_{10} x^3 = 30$, $x^3 = 10^{30}$, $x = 10^{10}$

23. $\log_{10} x = 5$, $x = 10^5$

24. $\ln 4x - \ln x^6 = \ln 2$, $\ln \dfrac{4}{x^5} = \ln 2$, $\dfrac{4}{x^5} = 2$, $x^5 = 2$, $x = \sqrt[5]{2}$

25. $\ln 2x^2 = \ln 3$, $2x^2 = 3$, $x^2 = 3/2$, $x = \sqrt{3/2}$ (we discard $-\sqrt{3/2}$ because it does not satisfy the original equation)

26. $\ln 3^x = \ln 2$, $x \ln 3 = \ln 2$, $x = \dfrac{\ln 2}{\ln 3}$

27. $\ln 5^{-2x} = \ln 3$, $-2x \ln 5 = \ln 3$, $x = -\dfrac{\ln 3}{2 \ln 5}$

28. $e^{-2x} = 5/3$, $-2x = \ln(5/3)$, $x = -\frac{1}{2}\ln(5/3)$

29. $e^{3x} = 7/2$, $3x = \ln(7/2)$, $x = \dfrac{1}{3}\ln(7/2)$

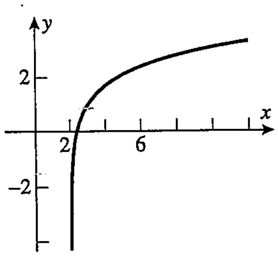
30. $e^x(1-2x) = 0$ so $e^x = 0$ (impossible) or $1 - 2x = 0$, $x = 1/2$

31. $e^{-x}(x+2) = 0$ so $e^{-x} = 0$ (impossible) or $x + 2 = 0$, $x = -2$

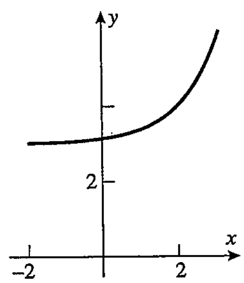
32. $e^{2x} - e^x - 6 = (e^x - 3)(e^x + 2) = 0$ so $e^x = -2$ (impossible) or $e^x = 3$, $x = \ln 3$

33. $e^{-2x} - 3e^{-x} + 2 = (e^{-x} - 2)(e^{-x} - 1) = 0$ so $e^{-x} = 2$, $x = -\ln 2$ or $e^{-x} = 1$, $x = 0$

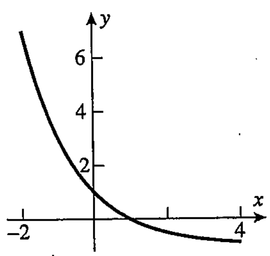
34. (a)



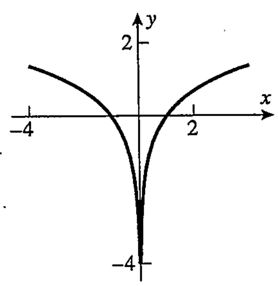
(b)



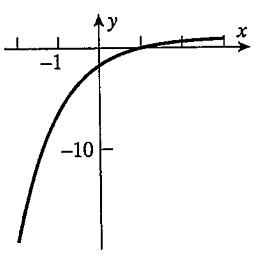
35. (a)



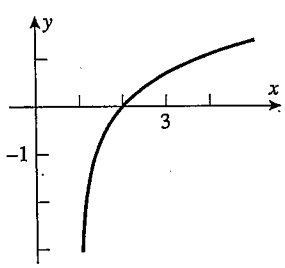
(b)



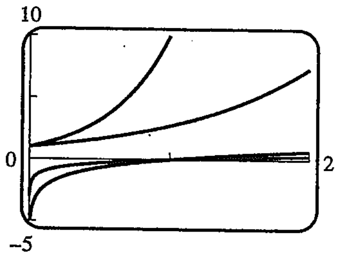
36. (a)

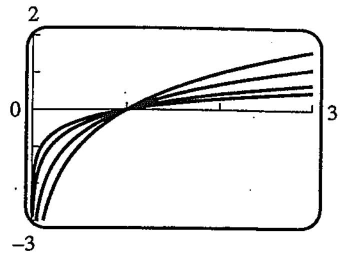


(b)



37. $\log_2 7.35 = (\log 7.35)/(\log 2) = (\ln 7.35)/(\ln 2) \approx 2.8777$;
$\log_5 0.6 = (\log 0.6)/(\log 5) = (\ln 0.6)/(\ln 5) \approx -0.3174$

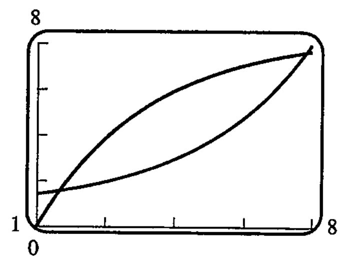
38. 

39. 

40. (a) Let $X = \log_b x$ and $Y = \log_a x$. Then $b^X = x$ and $a^Y = x$ so $a^Y = b^X$, or $a^{Y/X} = b$, which means $\log_a b = Y/X$. Substituting for Y and X yields $\dfrac{\log_a x}{\log_b x} = \log_a b, \log_b x = \dfrac{\log_a x}{\log_a b}$.

(b) Let $x = a$ to get $\log_b a = (\log_a a)/(\log_a b) = 1/(\log_a b)$ so $(\log_a b)(\log_b a) = 1$.
$(\log_2 81)(\log_3 32) = (\log_2[3^4])(\log_3[2^5]) = (4\log_2 3)(5\log_3 2) = 20(\log_2 3)(\log_3 2) = 20$

41. $x = y \approx 1.4710, x = y \approx 7.8571$



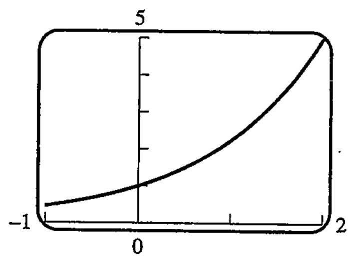
42. Since the units are billions, one trillion is 1,000 units. Solve $1000 = 0.051517(1.1306727)^x$ for x by taking common logarithms, resulting in $3 = \log 0.051517 + x \log 1.1306727$, which yields $x \approx 80.4$, so the debt first reached one trillion dollars around 1980.

43. (a) no, the curve passes through the origin

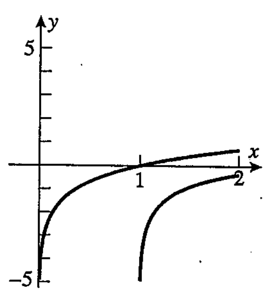
(b) $y = 2^{x/4}$

(c) $y = 2^{-x}$

(d) $y = (\sqrt{5})^x$



44. (a) As $x \to +\infty$ the function grows very slowly, but it is always increasing and tends to $+\infty$. As $x \to 1^+$ the function tends to $-\infty$.

(b) 

45. $\log(1/2) < 0$ so $3\log(1/2) < 2\log(1/2)$

46. Let $x = \log_b a$ and $y = \log_b c$, so $a = b^x$ and $c = b^y$.
First, $ac = b^x b^y = b^{x+y}$ or equivalently, $\log_b(ac) = x + y = \log_b a + \log_b c$.
Secondly, $a/c = b^x/b^y = b^{x-y}$ or equivalently, $\log_b(a/c) = x - y = \log_b a - \log_b c$.
Next, $a^r = (b^x)^r = b^{rx}$ or equivalently, $\log_b a^r = rx = r\log_b a$.
Finally, $1/c = 1/b^y = b^{-y}$ or equivalently, $\log_b(1/c) = -y = -\log_b c$.

47. $75e^{-t/125} = 15, t = -125\ln(1/5) = 125\ln 5 \approx 201$ days.

48. (a) If $t = 0$, then $Q = 12$ grams
    (b) $Q = 12e^{-0.055(4)} = 12e^{-0.22} \approx 9.63$ grams
    (c) $12e^{-0.055t} = 6, e^{-0.055t} = 0.5, t = -(\ln 0.5)/(0.055) \approx 12.6$ hours

49. (a) 7.4; basic (b) 4.2; acidic (c) 6.4; acidic (d) 5.9; acidic

50. (a) $\log[H^+] = -2.44, [H^+] = 10^{-2.44} \approx 3.6 \times 10^{-3}$ mol/L
    (b) $\log[H^+] = -8.06, [H^+] = 10^{-8.06} \approx 8.7 \times 10^{-9}$ mol/L

51. (a) 140 dB; damage (b) 120 dB; damage
    (c) 80 dB; no damage (d) 75 dB; no damage

52. Suppose that $I_1 = 3I_2$ and $\beta_1 = 10\log_{10} I_1/I_0$, $\beta_2 = 10\log_{10} I_2/I_0$. Then $I_1/I_0 = 3I_2/I_0$, $\log_{10} I_1/I_0 = \log_{10} 3I_2/I_0 = \log_{10} 3 + \log_{10} I_2/I_0$, $\beta_1 = 10\log_{10} 3 + \beta_2$, $\beta_1 - \beta_2 = 10\log_{10} 3 \approx 4.8$ decibels.

53. Let I_A and I_B be the intenisities of the automobile and blender, respectively. Then $\log_{10} I_A/I_0 = 7$ and $\log_{10} I_B/I_0 = 9.3$, $I_A = 10^7 I_0$ and $I_B = 10^{9.3} I_0$, so $I_B/I_A = 10^{2.3} \approx 200$.

54. The decibel level of the nth echo is $120(2/3)^n$;
$120(2/3)^n < 10$ if $(2/3)^n < 1/12, n < \dfrac{\log(1/12)}{\log(2/3)} = \dfrac{\log 12}{\log 1.5} \approx 6.13$ so 6 echoes can be heard.

55. (a) $\log E = 4.4 + 1.5(8.2) = 16.7, E = 10^{16.7} \approx 5 \times 10^{16}$ J
    (b) Let M_1 and M_2 be the magnitudes of earthquakes with energies of E and $10E$, respectively. Then $1.5(M_2 - M_1) = \log(10E) - \log E = \log 10 = 1$, $M_2 - M_1 = 1/1.5 = 2/3 \approx 0.67$.

56. Let E_1 and E_2 be the energies of earthquakes with magnitudes M and $M + 1$, respectively. Then $\log E_2 - \log E_1 = \log(E_2/E_1) = 1.5, E_2/E_1 = 10^{1.5} \approx 31.6$.

EXERCISE SET 1.7

1. The sum of the squares for the residuals for line I is approximately $1^2 + 1^2 + 1^2 + 0^2 + 2^2 + 1^2 + 1^2 + 1^2 = 10$, and the same for line II is approximately $0^2 + (0.4)^2 + (1.2)^2 + 0^2 + (2.2)^2 + (0.6)^2 + (0.2)^2 + 0^2 = 6.84$; line II is the regression line.

2. (a) The data appear to be periodic, so a trigonometric model may be appropriate.
   (b) The data appear to lie near a parabola, so a quadratic model may be appropriate.
   (c) The data appear to lie near a line, so a linear model may be appropriate.
   (d) The data appear to be randomly distributed, so no model seems to be appropriate.