CHAPTER 1
Functions

EXERCISE SET 1.1

1. (a) $-2.9, -2.0, 2.35, 2.9$ (b) none (c) $y = 0$
(d) $-1.75 \leq x \leq 2.15$ (e) $y_{\max} = 2.8$ at $x = -2.6$; $y_{\min} = -2.2$ at $x = 1.2$

2. (a) $x = -1, 4$ (b) none (c) $y = -1$
(d) $x = 0, 3, 5$ (e) $y_{\max} = 9$ at $x = 6$; $y_{\min} = -2$ at $x = 0$

3. (a) yes (b) yes
(c) no (vertical line test fails) (d) no (vertical line test fails)

4. (a) The natural domain of f is $x \neq -1$, and for g it is the set of all x. Whenever $x \neq$ $f(x) = g(x)$, but they have different domains.
(b) The domain of f is the set of all $x \geq 0$; the domain of g is the same.

5. (a) around 1943 (b) 1960; 4200
(c) no; you need the year's population (d) war; marketing techniques
(e) news of health risk; social pressure, antismoking campaigns, increased taxation

6. (a) around 1983 (b) 1966
(c) the former (d) no, it appears to be levelling out

7. (a) 1999, ~~$34,400~~ $43,400 (b) 1985, $37,000
(c) second year; graph has a larger (negative) slope

8. (a) In thousands, approximately $\dfrac{43.2 - 37.8}{6} = \dfrac{5.4}{6}$ per yr, or $900/yr
(b) The median income during 1993 increased from $37.8K to $38K (K for 'kilodollars'; all fig approximate). During 1996 it increased from $40K to $42K, and during 1999 it decre slightly from $43.2K to $43.1K. Thus the average rate of change measured on January 1 (40 - 37.8)/3 for the first three-yr period and (43.2 - 40)/3 for the second-year period, hence the median income as measured on January 1 increased more rapidly in the se three-year period. Measured on December 31, however, the numbers are (42 - 38)/3 and (- 42)/3, and the former is the greater number. Thus the answer to the question depend where in the year the median income is measured.
(c) 1993

9. (a) $f(0) = 3(0)^2 - 2 = -2; f(2) = 3(2)^2 - 2 = 10; f(-2) = 3(-2)^2 - 2 = 10; f(3) = 3(3)^2 - 2 =$
$f(\sqrt{2}) = 3(\sqrt{2})^2 - 2 = 4; f(3t) = 3(3t)^2 - 2 = 27t^2 - 2$
(b) $f(0) = 2(0) = 0; f(2) = 2(2) = 4; f(-2) = 2(-2) = -4; f(3) = 2(3) = 6; f(\sqrt{2}) = 2$
$f(3t) = 1/3t$ for $t > 1$ and $f(3t) = 6t$ for $t \leq 1$.

10. (a) $g(3) = \dfrac{3+1}{3-1} = 2;\ g(-1) = \dfrac{-1+1}{-1-1} = 0; g(\pi) = \dfrac{\pi+1}{\pi-1};\ g(-1.1) = \dfrac{-1.1+1}{-1.1-1} = \dfrac{-0.1}{-2.1} =$
$g(t^2 - 1) = \dfrac{t^2 - 1 + 1}{t^2 - 1 - 1} = \dfrac{t^2}{t^2 - 2}$
(b) $g(3) = \sqrt{3+1} = 2; g(-1) = 3; g(\pi) = \sqrt{\pi + 1}; g(-1.1) = 3;\ g(t^2 - 1) = 3$ if $t^2 < 2$
$g(t^2 - 1) = \sqrt{t^2 - 1 + 1} = |t|$ if $t^2 \geq 2$.

11. (a) $x \neq 3$ (b) $x \leq -\sqrt{3}$ or $x \geq \sqrt{3}$
(c) $x^2 - 2x + 5 = 0$ has no real solutions so $x^2 - 2x + 5$ is always positive or always negative. If $x = 0$, then $x^2 - 2x + 5 = 5 > 0$; domain: $(-\infty, +\infty)$.
(d) $x \neq 0$ (e) $\sin x \neq 1$, so $x \neq (2n + \frac{1}{2})\pi$, $n = 0, \pm 1, \pm 2, \ldots$

12. (a) $x \neq -\frac{7}{5}$
(b) $x - 3x^2$ must be nonnegative; $y = x - 3x^2$ is a parabola that crosses the x-axis at $x = 0, \frac{1}{3}$ and opens downward, thus $0 \leq x \leq \frac{1}{3}$
(c) $\dfrac{x^2 - 4}{x - 4} > 0$, so $x^2 - 4 > 0$ and $x - 4 > 0$, thus $x > 4$; or $x^2 - 4 < 0$ and $x - 4 < 0$, thus $-2 < x < 2$
(d) $x \neq -1$ (e) $\cos x \leq 1 < 2$, $2 - \cos x > 0$, all x

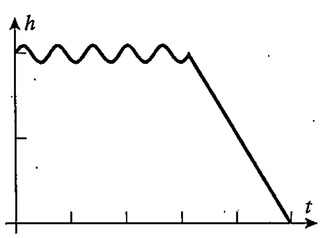
13. (a) $x \leq 3$ (b) $-2 \leq x \leq 2$ (c) $x \geq 0$ (d) all x (e) all x

14. (a) $x \geq \frac{2}{3}$ (b) $-\frac{3}{2} \leq x \leq \frac{3}{2}$ (c) $x \geq 0$ (d) $x \neq 0$ (e) $x \geq 0$

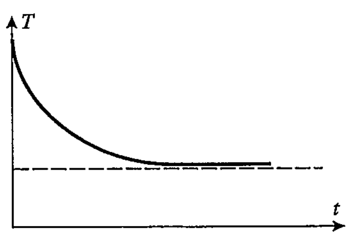
15. (a) Breaks could be caused by war, pestilence, flood, earthquakes, for example.
(b) C decreases for eight hours, takes a jump upwards, and then repeats.

16. (a) Yes, if the thermometer is not near a window or door or other source of sudden temperature change.
(b) No; the number is always an integer, so the changes are in movements (jumps) of at least one unit.

17.



18.



19. (a) $x = 2, 4$ (b) none (c) $x \leq 2$; $4 \leq x$ (d) $y_{\min} = -1$; no maximum value

20. (a) $x = 9$ (b) none (c) $x \geq 25$ (d) $y_{\min} = 1$; no maximum value

21. The cosine of θ is $(L - h)/L$ (side adjacent over hypotenuse), so $h = L(1 - \cos\theta)$.

22. The sine of $\theta/2$ is $(L/2)/10$ (side opposite over hypotenuse), so that $L = 20\sin(\theta/2)$.

23. (a) If $x < 0$, then $|x| = -x$ so $f(x) = -x + 3x + 1 = 2x + 1$. If $x \geq 0$, then $|x| = x$ so $f(x) = x + 3x + 1 = 4x + 1$;

$$f(x) = \begin{cases} 2x + 1, & x < 0 \\ 4x + 1, & x \geq 0 \end{cases}$$

(b) If $x < 0$, then $|x| = -x$ and $|x - 1| = 1 - x$ so $g(x) = -x + 1 - x = 1 - 2x$. If $0 \leq x < 1$, then $|x| = x$ and $|x - 1| = 1 - x$ so $g(x) = x + 1 - x = 1$. If $x \geq 1$, then $|x| = x$ and $|x - 1| = x - 1$ so $g(x) = x + x - 1 = 2x - 1$;

$$g(x) = \begin{cases} 1 - 2x, & x < 0 \\ 1, & 0 \leq x < 1 \\ 2x - 1, & x \geq 1 \end{cases}$$

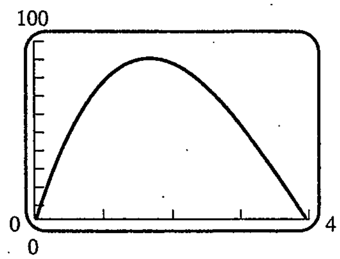
24. (a) If $x < 5/2$, then $|2x-5| = 5-2x$ so $f(x) = 3+(5-2x) = 8-2x$. If $x \geq 5/2$, then $|2x-5| = 2x-5$ so $f(x) = 3+(2x-5) = 2x-2$;

$$f(x) = \begin{cases} 8-2x, & x < 5/2 \\ 2x-2, & x \geq 5/2 \end{cases}$$

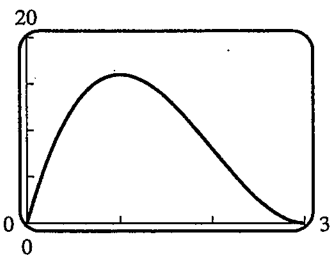
(b) If $x < -1$, then $|x-2| = 2-x$ and $|x+1| = -x-1$ so $g(x) = 3(2-x)-(-x-1) = 7-2x$. If $-1 \leq x < 2$, then $|x-2| = 2-x$ and $|x+1| = x+1$ so $g(x) = 3(2-x)-(x+1) = 5-4x$. If $x \geq 2$, then $|x-2| = x-2$ and $|x+1| = x+1$ so $g(x) = 3(x-2)-(x+1) = 2x-7$;

$$g(x) = \begin{cases} 7-2x, & x < -1 \\ 5-4x, & -1 \leq x < 2 \\ 2x-7, & x \geq 2 \end{cases}$$

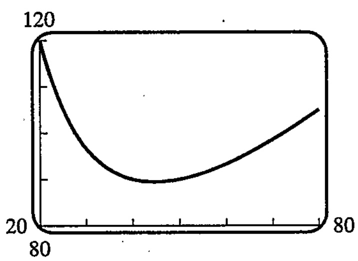
25. (a) $V = (8-2x)(15-2x)x$
(b) $0 \leq x \leq 4$
(c) $0 \leq V \leq 91$
(d) As x increases, V increases and then decreases; the maximum value could be approximated by zooming in on the graph.



26. (a) $V = (6-2x)^2 x$
(b) $0 < x < 3$
(c) $0 < V < 16$
(d) A x increases, V increases and then decreases; the maximum value occurs somewhere on $0 < x < 3$, and can be approximated by zooming with a graphing calculator.

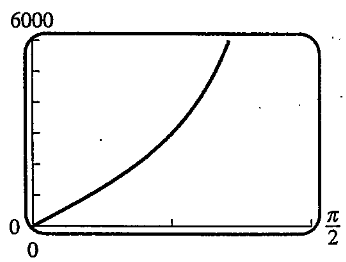


27. (a) The side adjacent to the building has length x, so $L = x + 2y$.
(b) $A = xy = 1000$, so $L = x + 2000/x$.
(c) all $x \neq 0$



(d) $L \approx 89.44$ ft

28. (a) $x = 3000 \tan\theta$
(b) $\theta \neq n\pi + \pi/2$ for any integer n, $-\infty < n < \infty$
(c) 3000 ft



29. (a) $V = 500 = \pi r^2 h$ so $h = \dfrac{500}{\pi r^2}$. Then

$$C = (0.02)(2)\pi r^2 + (0.01)2\pi r h = 0.04\pi r^2 + 0.02\pi r \frac{500}{\pi r^2}$$
$$= 0.04\pi r^2 + \frac{10}{r}; \; C_{\text{min}} \approx 4.39 \text{ cents at } r \approx 3.4 \text{ cm},$$
$h \approx 13.8$ cm