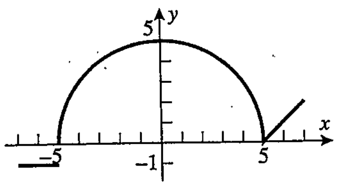
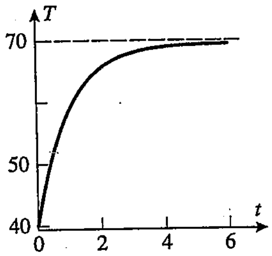
REVIEW EXERCISES, CHAPTER 1

1.



2. (a) $f(-2) = 2$, $g(3) = 2$ (b) $x = -3, 3$ (c) $x < -2, x > 3$
   (d) the domain is $-5 \le x \le 5$ and the range is $-5 \le y \le 4$
   (e) the domain is $-4 \le x \le 4.1$, the range is $-3 \le y \le 5$
   (f) $f(x) = 0$ at $x = -3, 5$; $g(x) = 0$ at $x = -3, 2$

3.



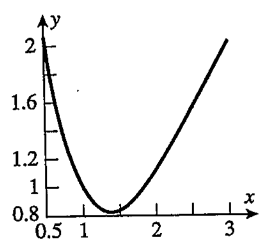
4. Assume that the paint is applied in a thin veneer of uniform thickness, so that the quantity of paint to be used is proportional to the area covered. If P is the amount of paint to be used, $P = k\pi r^2$. The constant k depends on physical factors, such as the thickness of the paint, absorption of the wood, etc.

5. (a) If the side has length x and height h, then $V = 8 = x^2h$, so $h = 8/x^2$. Then the cost $C = 5x^2 + 2(4)(xh) = 5x^2 + 64/x$.
   (b) The domain of C is $(0, +\infty)$ because x can be very large (just take h very small).

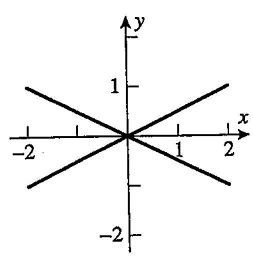
6. (a) Suppose the radius of the uncoated ball is r and that of the coated ball is $r + h$. Then the plastic has volume equal to the difference of the volumes, i.e. $V = \frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi h[3r^2+3rh+h^2]$ in^3. But $r = 3$ and hence $V = \frac{4}{3}\pi h[27+9h+h^2]$.
   (b) $0 < h < \infty$

7. (a) The base has sides $(10 - 2x)/2$ and $6 - 2x$, and the height is x, so $V = (6 - 2x)(5 - x)x$ ft^3.
   (b) From the picture we see that $x < 5$ and $2x < 6$, so $0 < x < 3$.
   (c) 3.57 ft ×3.79 ft ×1.21 ft

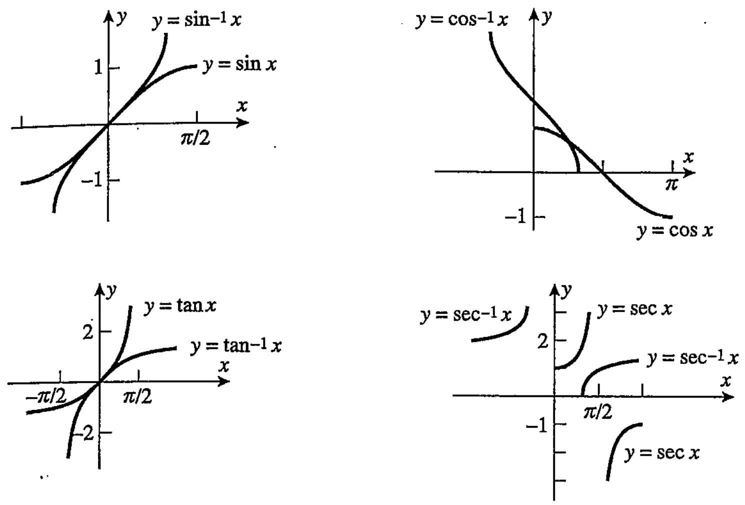
8. (a) $d = \sqrt{(x-1)^2 + 1/x^2}$;
   (b) $-\infty < x < 0$, $0 < x < +\infty$
   (c) $d \approx 0.82$ at $x \approx 1.38$



9.



(b)



27. (a) $x = f(y) = 8y^3 - 1$; $y = f^{-1}(x) = \left(\dfrac{x+1}{8}\right)^{1/3} = \dfrac{1}{2}(x+1)^{1/3}$

(b) $f(x) = (x-1)^2$; f does not have an inverse because f is not one-to-one, for example $f(0) = f(2) = 1$.

(c) $x = f(y) = (e^y)^2 + 1$; $y = f^{-1}(x) = \ln\sqrt{x-1} = \frac{1}{2}\ln(x-1)$

(d) $x = f(y) = \dfrac{y+2}{y-1}$; $y = f^{-1}(x) = \dfrac{x+2}{x-1}$

(e) $x = f(y) = \sin\left(\dfrac{1-2y}{y}\right)$; $y = \dfrac{1}{2+\sin^{-1}x}$

(f) $x = \dfrac{1}{1+3\tan^{-1}y}$; $y = \tan\left(\dfrac{1-x}{3x}\right)$

28. It is necessary and sufficient that the graph of f pass the horizontal line test. Suppose to the contrary that $\dfrac{ah+b}{ch+d} = \dfrac{ak+b}{ck+d}$ for $h \neq k$. Then $achk+bck+adh+bd = achk+adk+bch+bd$, $bc(h-k) = ad(h-k)$. It follows from $h \neq k$ that $ad - bc = 0$. These steps are reversible, hence f^{-1} exists if and only if $ad - bc \neq 0$, and if so, then

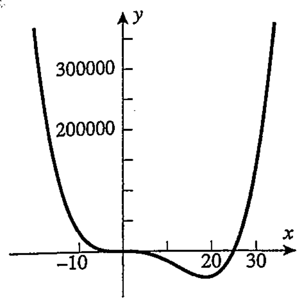
$$x = \frac{ay+b}{cy+d},\ xcy + xd = ay + b,\ y(cx-a) = b - xd,\ y = \frac{b-xd}{cx-a} = f^{-1}(x)$$

29. Draw equilateral triangles of sides 5, 12, 13, and 3, 4, 5. Then $\sin[\cos^{-1}(4/5)] = 3/5$, $\sin[\cos^{-1}(5/13)] = 12/13$, $\cos[\sin^{-1}(4/5)] = 3/5$, $\cos[\sin^{-1}(5/13)] = 12/13$

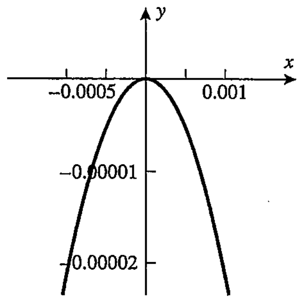
(a)
$$\begin{aligned}\cos[\cos^{-1}(4/5) + \sin^{-1}(5/13)] &= \cos(\cos^{-1}(4/5))\cos(\sin^{-1}(5/13)) \\ &\quad - \sin(\cos^{-1}(4/5))\sin(\sin^{-1}(5/13)) \\ &= \frac{4}{5}\frac{12}{13} - \frac{3}{5}\frac{5}{13} = \frac{33}{65}.\end{aligned}$$

(b)
$$\begin{aligned}\sin[\sin^{-1}(4/5) + \cos^{-1}(5/13)] &= \sin(\sin^{-1}(4/5))\cos(\cos^{-1}(5/13)) \\ &\quad + \cos(\sin^{-1}(4/5))\sin(\cos^{-1}(5/13)) \\ &= \frac{4}{5}\frac{5}{13} + \frac{3}{5}\frac{12}{13} = \frac{56}{65}.\end{aligned}$$

10. (a) On the interval $[-20, 30]$ the curve seems tame,



(b) but seen close up on the interval $[-1/1000, +1/1000]$ we see that there is some wiggling near the origin.



11.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	0	-1	2	1	3	-2	-3	4	-4
$g(x)$	3	2	1	-3	-1	-4	4	-2	0
$(f \circ g)(x)$	4	-3	-2	-1	1	0	-4	2	3
$(g \circ f)(x)$	-1	-3	4	-4	-2	1	2	0	3

12. $f \circ g(x) = -1/|x|$ with domain $x \neq 0$, and $g \circ f(x)$ is nowhere defined, with domain $\emptyset$.

13. $f(g(x)) = (3x+2)^2+1, g(f(x)) = 3(x^2+1)+2$, so $9x^2+12x+5 = 3x^2+5, 6x^2+12x = 0, x = 0, -2$

14. (a) $(3-x)/x$

(b) no; the definition of $f(g(x))$ requires $g(x)$ to be defined, so $x \neq 1$, and $f(g(x))$ requires $g(x) \neq -1$, so we must have $g(x) \neq -1$, i.e. $x \neq 0$; whereas $h(x)$ only requires $x \neq 0$

15. When $f(g(h(x)))$ is defined, we require $g(h(x)) \neq 1$ and $h(x) \neq 0$. The first requirement is equivalent to $x \neq \pm 1$, the second is equivalent to $x \neq \pm\sqrt(2)$. For all other $x, f \cdot g \cdot h = 1/(2-x^2)$.

16. $g(x) = x^2 + 2x$

17. (a) even $\times$ odd = odd

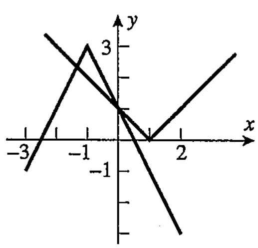
(b) a square is even

(c) even + odd is neither

(d) odd $\times$ odd = even

18. (a) $y = |x-1|$, $y = |(-x)-1| = |x+1|$,
$y = 2|x+1|$, $y = 2|x+1| - 3$,
$y = -2|x+1| + 3$

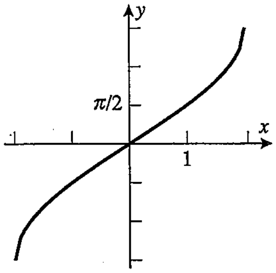
(b)



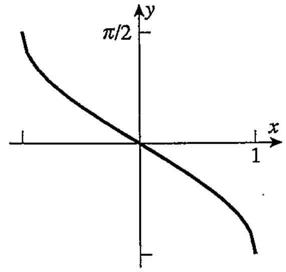
19. (a) The circle of radius 1 centered at (a, a^2); therefore, the family of all circles of radius 1 with centers on the parabola $y = x^2$.

(b) All parabolas which open up, have latus rectum equal to 1 and vertex on the line $y = x/2$.

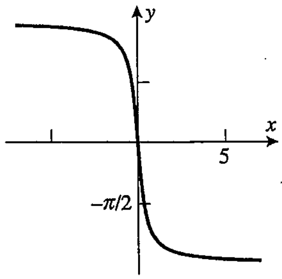
30. (a)



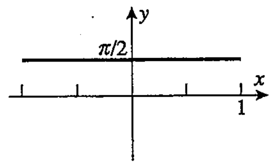
(b)



(c)



(d)



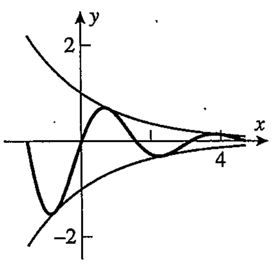
31. $y = 5$ ft $= 60$ in, so $60 = \log x$, $x = 10^{60}$ in $\approx 1.58 \times 10^{55}$ mi.

32. $y = 100$ mi $= 12 \times 5280 \times 100$ in, so $x = \log y = \log 12 + \log 5280 + \log 100 \approx 6.8018$ in

33. $3\ln\left(e^{2x}(e^x)^3\right) + 2\exp(\ln 1) = 3\ln e^{2x} + 3\ln(e^x)^3 + 2\cdot 1 = 3(2x) + (3\cdot 3)x + 2 = 15x + 2$

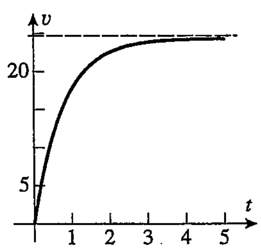
34. $Y = \ln(Ce^{kt}) = \ln C + \ln e^{kt} = \ln C + kt$, a line with slope k and Y-intercept $\ln C$

35. (a)



(b) The curve $y = e^{-x/2}\sin 2x$ has x–intercepts at $x = -\pi/2, 0, \pi/2, \pi, 3\pi/2$. It intersects the curve $y = e^{-x/2}$ at $x = \pi/4, 5\pi/4$ and it intersects the curve $y = -e^{-x/2}$ at $x = -\pi/4, 3\pi/4$.

36. (a)



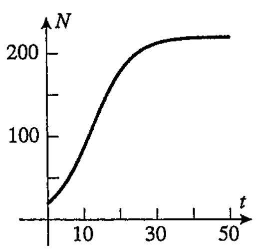
(b) $\lim_{t\to\infty}(1 - e^{-1.3t}) = 1$,
and thus as $t \to 0$, $v \to 24.61$ ft/s.

(c) For large t the velocity approaches $c = 24.61$.

(d) No; but it comes very close (arbitrarily close).

(e) 3.009 s

37. (a)



(b) $N = 80$ when $t = 9.35$ yrs

(c) 220 sheep

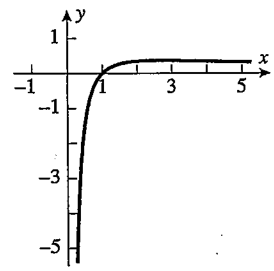
38. (a) The potato is done in the interval $27.65 < t < 32.71$.

(b) 91.54 min. The oven temperature is always 400° F, so the difference between the oven temperature and the potato temperature is $D = 400 - T$. Initially $D = 325$, so solve $D = 75 + 325/2 = 237.5$ for t, $t \approx 22.76$.

39. (a) The function $\ln x - x^{0.2}$ is negative at $x = 1$ and positive at $x = 4$, so it is reasonable to expect it to be zero somewhere in between. (This will be established later in this book.)

(b) $x = 3.654$ and 3.32105×10^5

40. (a) If $x^k = e^x$ then $k \ln x = x$, or $\dfrac{\ln x}{x} = \dfrac{1}{k}$. The steps are reversible.



(b) By zooming it is seen that the maximum value of y is approximately 0.368 (actually, $1/e$), so there are two distinct solutions of $x^k = e^x$ whenever $k > 1/0.368 \approx 2.717$.

(c) $x \approx 1.155$

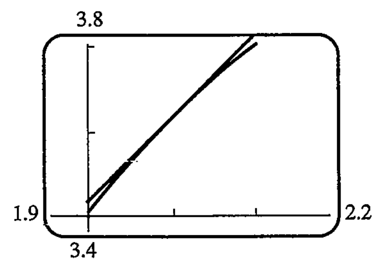
41. (a)

1.90	1.92	1.94	1.96	1.98	2.00	2.02	2.04	2.06	2.08	2.10
3.4161	3.4639	3.5100	3.5543	3.5967	3.6372	3.6756	3.7119	3.7459	3.7775	3.8068

(b) $y = 1.9589x - 0.2910$

(c) $y - 3.6372 = 1.9589(x - 2)$, or $y = 1.9589x - 0.2806$

(d) As one zooms in on the point $(2, f(2))$ the two curves seem to converge to one line.



42. (a)

−0.10	−0.08	−0.06	−0.04	−0.02	0.00	0.02	0.04	0.06	0.08	0.10
0.9950	0.9968	0.9982	0.9992	0.9998	1.0000	0.9998	0.9992	0.9982	0.9968	0.9950

(b) $y = 13.669x^2 + 2.865x + 0.733$

(c) $y = -\frac{1}{2}x^2 + 1$

(d) As one zooms in on the point $(0, f(0))$ the two curves seem to converge to one curve.