Test 2: Basic Calculus I

Math 110 Fall 2007 Thursday November 1
 7:00pm

Name: _____

Directions: Read *all* problems first before answering any of them. There are 7 pages in this test. This test is intended to be taken in 55-minutes. You may have as much time as you like (within reason!) **You may use a calculator.** You must show all relevant work to support your answers. Use complete English sentences and CLEARLY indicate your final answers to be graded from your "scratch work."

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score	Maximum
1		20
2		20
3		20
4		20
5		20
BONUS		10
Total		**100**

1. Differentiation Rules. *20 points.* Evaluate the following derivatives, making sure to clearly specify all the derivative rules you use to find your answer. Do NOT simplify your answers.

RULE 1: $(f+g)' = f' + g'$ RULE 2: $(f-g)' = f' + g'$ RULE 3: $(cf)' = cf'$

RULE 4: $(fg)' = f'g + fg'$ RULE 5: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ RULE 6: $(f(g))' = f'(g)g'$

a. *(5 points.)* $\dfrac{d}{dx}[x^7 - \sqrt{9} + \dfrac{25}{x^{1/5}} + e^3]$

b. *(5 points.)* $\dfrac{d}{dr}[\sin(r)\cos(r)\tan(r)]$.

c. *(5 points.)* $\dfrac{d}{dy}\left[\dfrac{4^y + 5}{4y + 5}\right]$

d. *(5 points.)* $\dfrac{d}{ds}[e^{\ln(\sqrt{s}+1)}]$

2. TRUE OR FALSE: Chain Rule, Differentials, Differentiability, Continuity, Rrelated Rates. *20 points.*

TRUE or FALSE: put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE one way to prove this is to show there exists a counterexample and use it to prove the statement is FALSE (at least once).

(a) *5 points.* **TRUE or FALSE?** "If $f(x) = -f(-x)$ for all x, then $f'(x) = f'(-x)$ for all x."

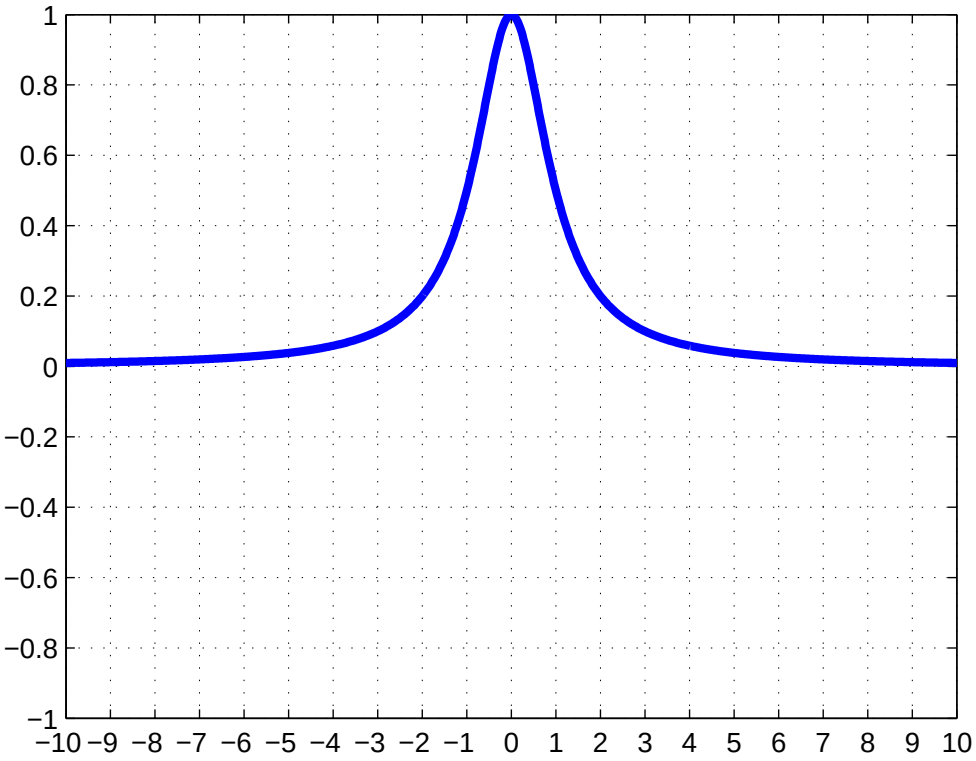
(b) *5 points.* **TRUE or FALSE?** "If the length of one side of a square increases by an amount dx, the area of the square will increase by an amount twice as large, $2\,dx$."

(c) *5 points.* **TRUE or FALSE?** "If the entire graph of a function can be drawn without picking up the drawing implement at any point then that function is differentiable everywhere."

(d) *5 points.* **TRUE or FALSE?** "If at $t = 3$ $\dfrac{dy}{dt} = 4$ snarfs per minute and $\dfrac{dx}{dt} = -2$ Muggles per minute, then $\dfrac{dy}{dx} = -\dfrac{1}{2}$ Muggles per snarf at $t = 3$."

3. Visualization of Derivative Function. *20 points.* Consider the graph of the following function $f(x)$. This problem is about interpreting the derivative function graphically and conceptually.

(a) (*10 points.*) On the same axes as $f(x)$ below, sketch a graph of $f'(x)$.



(b) (*10 points.*) Write down a paragraph explaining all of the particular notable features of the graph of $f'(x)$ you drew, providing the reasons for including these features in your graph. For example, notable features of the given graph $f(x)$ are that it has a single horizontal asymptote as $x \to \infty$ or $x \to -\infty$ and possesses a single maximum value at $x = 0$ and is always positive. You should describe and explain the notable features of your sketch of the graph of $f'(x)$ similarly.

4. Definition of Derivative, Limits. *20 points.*

Consider the following limit

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1.$$

(a) (*10 points.*) Interpret the above limit as the definition of the derivative of some function $f(x)$ evaluated at some point a. Determine $f(x)$, a and confirm that the value of the limit above is indeed equal to $f'(a)$ for your choice of $f(x)$ and a.

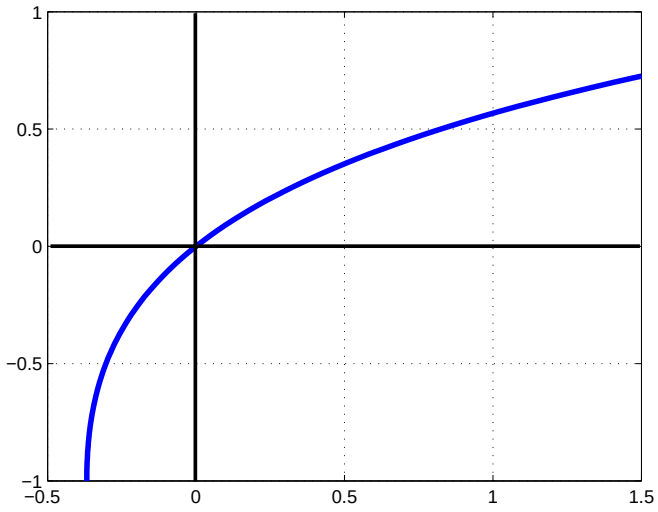
(b) (*10 points.*) Use the limit definition of the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

and the value of the limit at the top of the page to prove that $f'(x) = f(x)$ when $f(x) = e^x$. Be careful to clearly state what limit rules you are applying during each step of your proof that $f'(x) = f(x)$ when $f(x) = e^x$.

5. Chain Rule, Tangent Lines, Implicit Differentiation, Logarithmic Differentiation. *20 points.*

Consider the graph of the implicit curve $x = ye^y$ given below



(a) (*10 points.*) Show that $\dfrac{dy}{dx} = \dfrac{y}{x}\dfrac{1}{y+1}$.

(b) (*10 points.*) Find the equation of the tangent line to the implicitly defined curve $x = ye^y$ at the point $(0, 0)$. If the tangent line does not exist at that point, explain why.

BONUS QUESTION. *10 points.* Although the function $y = f(x)$ implicitly defined through the equation $x = ye^y$ can not be found explicitly, you found its derivative in Question 4. For what values of y will the inverse function $x = f^{-1}(y)$ exist? For what values of x is the function $y = f(x)$ invertible? If possible, find an explicit formula for the inverse of the implicitly defined function $y = f(x)$ from Question 4? EXPLAIN YOUR ANSWER. Regardless of whether you can find $f^{-1}(y)$ explicitly, can you obtain its derivative explicitly? If so, write down a formula for it.

OR

Use a local linear approximation to the implicitly defined function $x = ye^y$ near $(0, 0)$ to approximate the solution to the equation $0.5 = ye^y$. Draw a picture to indicate whether your approximation is greater than or less than the actual exact answer and discuss how you would improve your estimated value.