Report on Test 1

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Grade Distribution (N=67)

Range	99+	92+	90+	87+	82+	80+	75+	70+	64+	60+	55+	50+	50-
Grade	A+	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	4	6	4	2	9	2	5	9	7	3	5	4	7

Summary Overall class performance was pretty good. There were more than twice as many A's as F's! The mean score was 72, the median score was 74 and the mode was 70 and 94. The high score was a 104. The low score was 11.

#1 Logarithms, Exponentials, Piecewise-Defined Functions. Ahh, Blogarithms! This questions is really about how well do you understand that the natural exponential function and natural logarithm function are defined as inverses of each other. **(a)** Is $\text{blog}(e^x) = x$ for every value of x? **YES!** Why? Because $\text{blog}(e^x) = \ln(e^x)$ since we know (or can see from the graph given in part (c) that $e^x > 0$ for every x). Then we know that $\ln(e^x) = x$ for every x value, so $\text{blog}(e^x) = x$. There's no need to check on whether x is > 0, < 0 or $= 0$ in this section because for all of those values e^x outputs a positive number and it is THAT number that you are taking the blog of. **(b)** Is $e^{\text{blog}(x)} = x$ for every value of x? **NO!** When $x > 0$ blogarithms are identical to logarithms, so for those positive values of x, $e^{\text{blog}(x)} = e^{\ln(x)} = x$. However, when $x = 0$, $e^{\text{blog}(0)} = e^0 = 1 \neq 0$. You could stop there, because you have found a value of x where the statement $e^{\text{blog}(x)} = x$ is FALSE. However, let's also consider $x < 0$. In that case, $e^{\text{blog}(x)} = e^{\ln(-x)} = -x$ (we can still do this operation because as long as $x < 0$, then $-x > 0$ so $\ln(-x)$ is defined.) Clearly, $-x \neq x$ so the statement $e^{\text{blog}(x)} = x$ is false for $x \leq 0$. **(c)** The graph of $\text{blog}(x)$ is taken in pieces. It must have a solid dot at $(0, 0)$ and when $x > 0$ it looks identical to $\ln(x)$ which of course is the graph e^x reflected about the line $y = x$ since e^x and $\ln(x)$ are inverses of each other. For $x < 0$ $\text{blog}(x) = \ln(-x)$ which is equivalent to reflecting the graph of $\ln(x)$ about the y-axis. You should also be careful to make sure that your graph of blogarithm has a vertical asymptote at $x = 0$ and crosses the x-axis at $(-1, 0)$ and $(1, 0)$.

#2 Limits, Continuity. This question is about inverting your usual thinking about a problem. Here, YOU have to provide functions and values which have specific limits as your function's input approaches those values instead of providing an output limit given a function and an input value. **(a)** A one-sided limit looks like $\lim_{x\to 0^-}$ or $\lim_{x\to 0^+}$ if you select $a = 0$. In order for a one-sided limit not to exist the output values must either become infinitely large, or the function is undefined at many places near $x = a$ or the limit must oscillate violently and never settle down to a limit. Examples are (respectively), $\lim_{x\to 0^-} \frac{1}{x}$, $\lim_{x\to 0^-} \sqrt{x}$ or $\lim_{x\to 0^-} \sin(\frac{1}{x})$
(b) A **limit of indeterminate form** is one in which initially it appears as if the limit equals $\frac{\infty}{\infty}$ or $\frac{0}{0}$. To evaluate such limits one must usually manipulate the function algebraically in some way and thus produce a more definitive answer (which could be DOES NOT EXIST) but in this case was supposed to be zero. One example is $\lim_{x\to\infty} \frac{x^2+1}{x^3+2} = \frac{\infty}{\infty} \approx \lim_{x\to\infty} \frac{x^2}{x^3} = \lim_{x\to\infty} \frac{1}{x} = 0$. Another example is $\lim_{x\to 0} \frac{1-\cos(x)}{x} = \frac{0}{0} = 0$.
(c) A two sided limit of a non-constant function whose value is equal to 2 has many many possible answers. Some are $\lim_{x\to 0} x+2$ or $\lim_{x\to\infty} \frac{1}{x}+2$. **(d)** In this problem the idea was to choose two *related* functions $f(x)$ and $g(x)$ which are practically identical except $g(x)$ is continuous at $x = 0$ and the other function $f(x)$ has a removable discontinuity at $x = 0$. In order to have a function $f(x)$ which has such a removable discontinuity you must think of a function for which $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) \neq f(0)$ and then define $g(x)$ such that for all values $x \neq 0$, $g(x) = f(x)$ and when $x = 0$, $g(x) = \lim_{x\to 0} f(x)$. An example is $f(x) = \frac{x^2}{x}$.

#3 Functions, Inverses. This question is a riff on Question 11 on Page 97 of the Chapter 1 Review Exercises in *Anton, Bivens & Davis.* However, given the extra information that the entire domain of $f(x)$ is the set $\{-2, -1, 0, 1, 2\}$ which is a discrete set of numbers one has to be careful of applying ideas from continuous variables. The horizontal line test does NOT apply while reflecting the graph around the line $y = x$ DOES. However, the basic idea that an inverse of a function reverses what the original function does allows one to see that whenever $f^{-1}(x) = f(x)$ it is clear that $f(f(x)) = x$ and $f^3(x) = f(x)$. In order to prove a function is involutory, $f^{-1}(x) = f(x)$ must be true for every value of x in the domain, not just specific x values, but since there are just 5 elements in the domain, it's relatively easy to do just that.