MATH 110 Exam 1

September 30, 2004

Name: Key

Please write up your solutions in the space provided. You may use the back of the pages for scratch work and for extra space if you need it. If you wish me to grade any work on the backs of pages, please draw attention to it on the front pages. Boxed answers are appreciated for numerical problems. Calculators are permitted for this exam. *Note: To receive full credit, your answer must be exact (not approximate) and you must provide appropriate justification.*

Be sure to carefully read the instructions for each problem.

There are 5 problems worth 60 points.

Good luck!

Question	Points	Score
1	10	
2	15	
3	10	
4	10	
5	15	
EXTRA	4	
Total	60	

1. (a) Solve $e^{2x+3} - 7 = 0$ for x.

$$e^{2x+3} = 7$$

$$2x+3 = \ln(7)$$

$$2x = \ln(7) - 3$$

$$x = \frac{\ln(7) - 3}{2}$$

(b) Find $\sin\theta$ given that $0 < \theta < \frac{\pi}{2}$ and $\cos\theta = \frac{1}{5}$.

$$\cos\theta = \frac{1}{5}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$= 1 - \left(\frac{1}{5}\right)^2$$

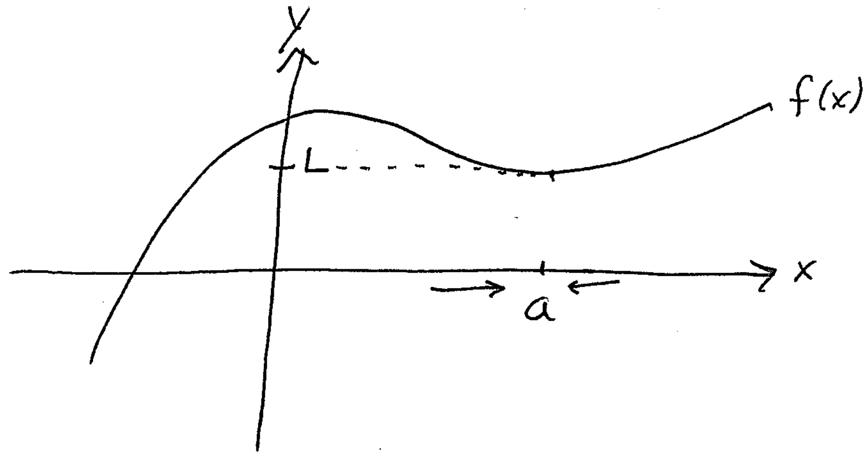
$$= 1 - \frac{1}{25}$$

$$= \frac{24}{25}$$

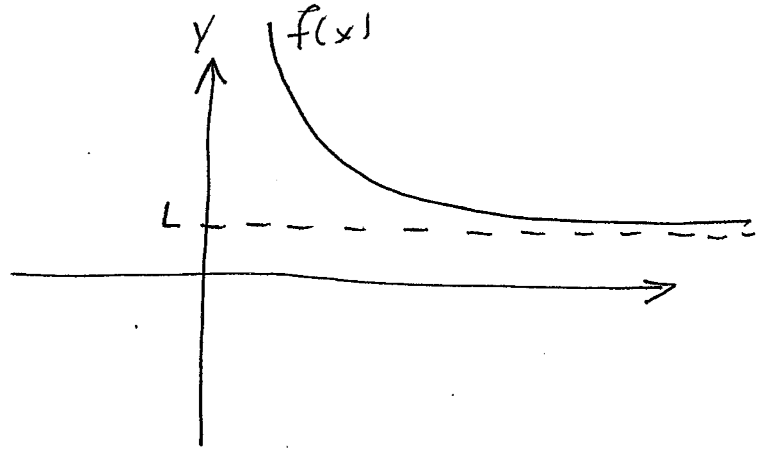
$$\sin\theta = \sqrt{\frac{24}{25}} = \frac{2\sqrt{6}}{5}$$

2. Illustrate each of the following statements with a sketch.

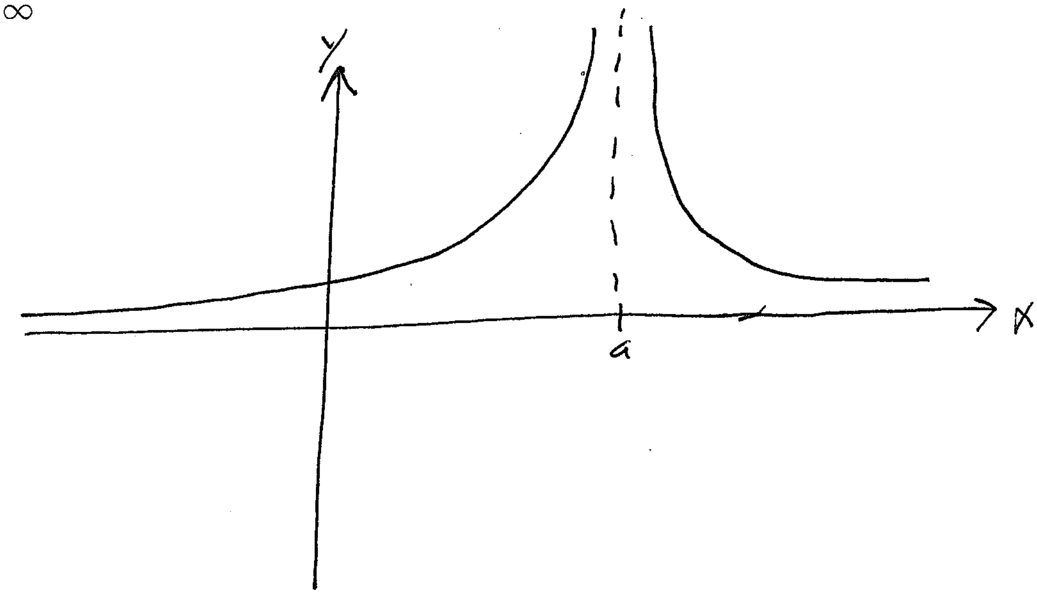
(a) $\lim_{x \to a} f(x) = L$



(b) $\lim_{x \to \infty} f(x) = L$



(c) $\lim_{x \to a} f(x) = \infty$



3. Let $f(x) = \dfrac{3x^2 - x - 2}{x^2 + 1}$.

(a) What is the domain of f?

All $x \in \mathbb{R}$.

$x^2 + 1 \neq 0$ for all x

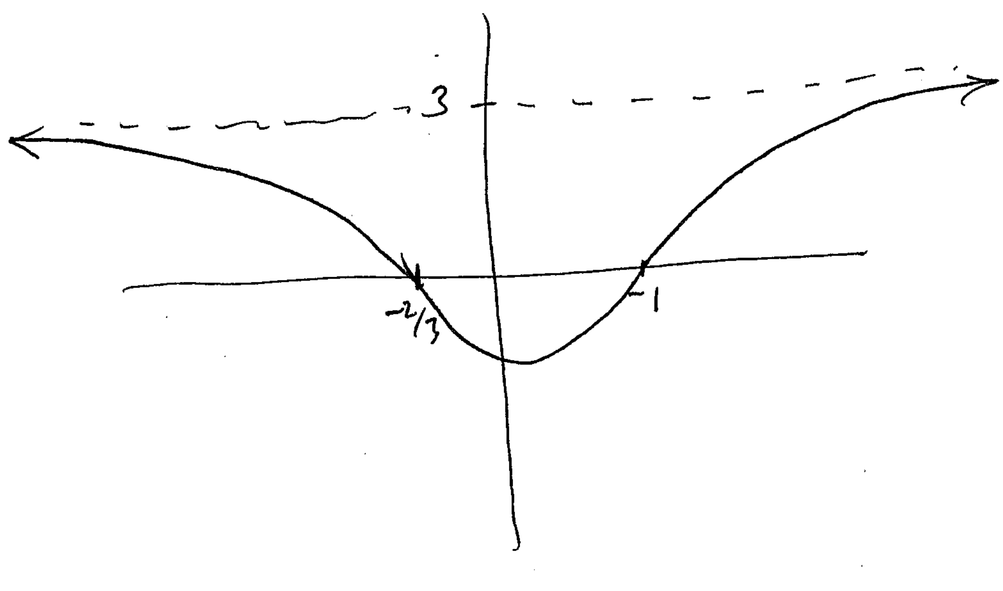
(b) Use a graph to discover all asymptotes. Then use properties of limits to prove what you have discovered.

$$\lim_{x \to \infty} \frac{3x^2 - x - 2}{x^2 + 1} = \lim_{x \to \infty} \frac{3x^2}{x^2} = 3$$

$$\lim_{x \to -\infty} \frac{3x^2 - x - 2}{x^2 + 1} = \lim_{x \to -\infty} \frac{3x^2}{x^2} = 3$$

No vertical asymptotes

One horizontal asymptote at $y = 3$



$3x^2 - x - 2 = 0 \Rightarrow (3x+2)(x-1)$

4. Determine the following limits. Answer with a number, $-\infty$, ∞, or "does not exist" **and** provide a clear justification for your answer. In order to receive full credit, you must justify your answer using properties of limits. However, partial credit will be given for answers supported solely by graphical or numerical evidence.

(a) $\lim\limits_{x\to 3} \dfrac{x^2-9}{x^2+2x-3}$ $= \lim\limits_{x\to 3} \dfrac{(x-3)(x+3)}{(x-1)(x+3)} = \lim\limits_{x\to 3} \dfrac{x-3}{x+3} = 0$

$\left(\lim \left(\dfrac{f}{g}\right) = \dfrac{\lim f}{\lim g} \text{ if } \lim g \neq 0 \right)$

(b) $\lim\limits_{x\to 0} \dfrac{1-\sqrt{1-3x}}{x}$

$$= \lim_{x\to 0} \frac{(1+\sqrt{1-3x})}{(1+\sqrt{1-3x})} \frac{(1-\sqrt{1-3x})}{x}$$

$$= \lim_{x\to 0} \frac{1^2 - (1-3x)}{(1+\sqrt{1-3x})\,x}$$

$$= \lim_{x\to 0} \frac{1-1+3x}{(1+\sqrt{1-3x})\,x}$$

$$= \lim_{x\to 0} \frac{3x}{(1+\sqrt{1-3x})\,x}$$

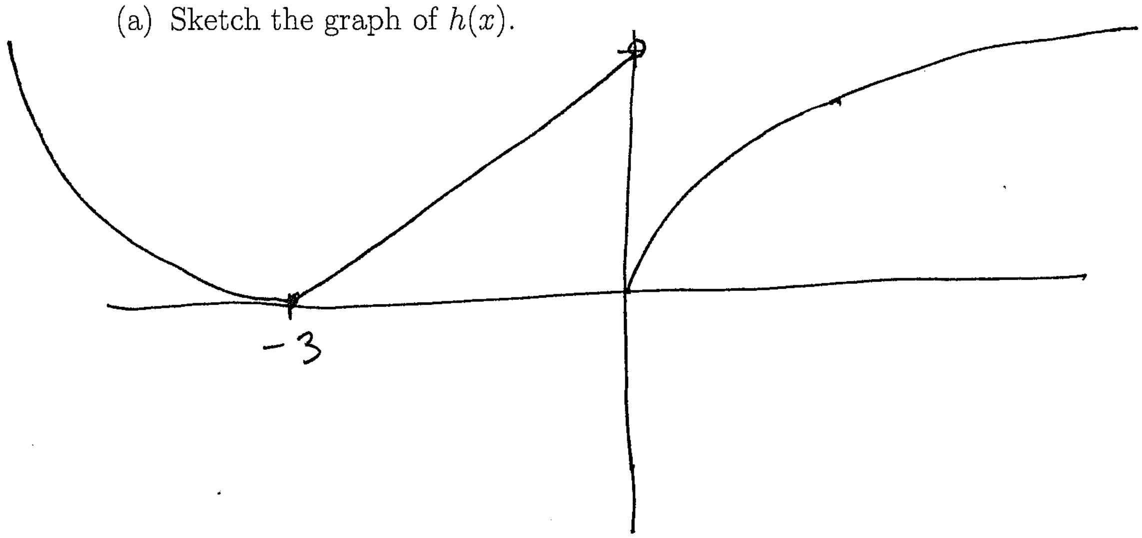
$$= \lim_{x\to 0} \frac{3}{1+\sqrt{1-3x}} = \frac{3}{1+\sqrt{1}}$$

$$= \lim 3 = \frac{3}{2}$$

5. Let h be defined by

$$h(x) = \begin{cases} (x+3)^2 & \text{if } x < -3, \\ x+3 & \text{if } -3 \le x \le 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$$

(a) Sketch the graph of $h(x)$.



(b) Using the definition of continuity, explain why h is discontinuous at $x = 0$.

$\lim_{x \to 0^-} h(x) = 3 \neq \lim_{x \to 0^+} h(x) = 0 \Rightarrow \lim_{x \to 0^+} h(x)$ does not exist, so $h(0) = 3$ is not equal to $\lim_{x \to 0} h(x)$.

(c) Is the discontinuity at $x = 0$ removable? If so, redefine $h(0)$ so the discontinuity is removed. If not, explain why the discontinuity is not removable.

The discontinuity is NOT removable. However you define the function $h(x)$ at $x = 0$, either equal to zero or 3, it will not equal one of the one sided limits

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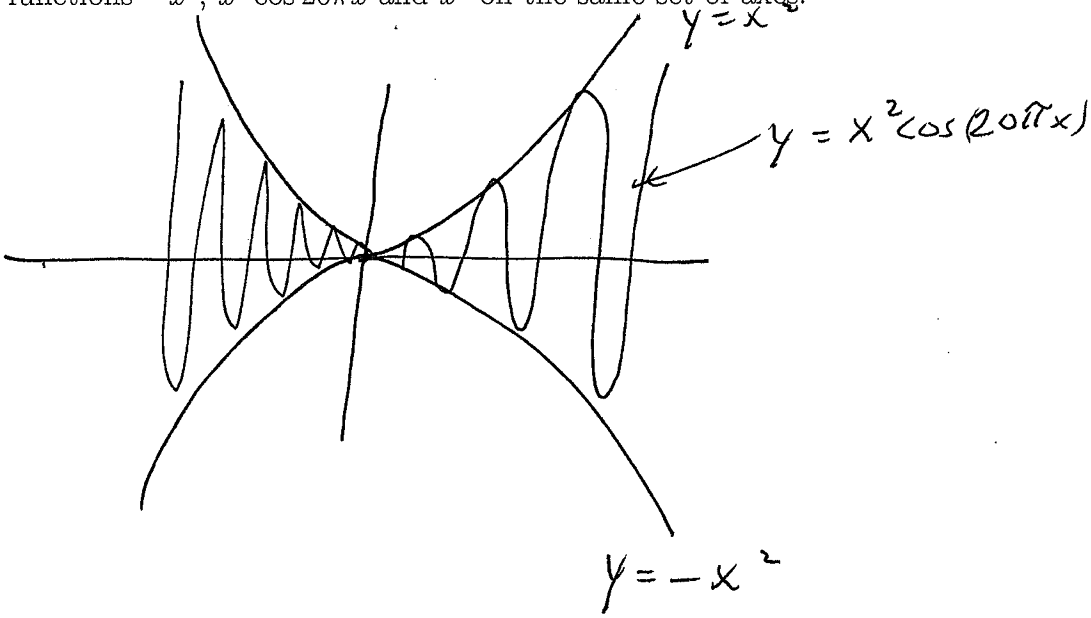
EXTRA CREDIT: Use the Squeeze Theorem to show that $\lim_{x \to 0} x^2 \cos 20\pi x = 0$. Illustrate by graphing the functions $-x^2$, $x^2 \cos 20\pi x$ and x^2 on the same set of axes.



$$-x^2 < x^2 \cos(20\pi x) < x^2$$

$$\lim_{x \to 0} (-x^2) = 0$$

$$\lim_{x \to 0} x^2 = 0$$

Therefore $0 < \lim_{x \to 0} x^2 \cos(20\pi x) < 0$.

i.e. $\lim_{x \to 0} x^2 \cos(20\pi x) = 0$