MATH 110 Exam 3

November 18, 2004

Name: Key

Please write up your solutions in the space provided. You may use the back of the pages for scratch work and for extra space if you need it. If you wish me to grade any work on the backs of pages, please draw attention to it on the front pages. Boxed answers are appreciated for numerical problems. Although calculators are permitted for this exam, *you must provide appropriate justification for your final answers.* Please carefully read the instructions for each problem.

There are 5 problems worth 60 points and 1 extra credit problem worth 3 points.

Good luck!

Question	Points	Score
1	15	
2	10	
3	15	
4	10	
5	10	
EXTRA	3	
Total	60	

1. For each item below, find $y'(x)$. Then find a value for $y'(0)$. SHOW ALL WORK.

(a) $y = (3x^2 + 1)^{100}$

$$\frac{dy}{dx} = 100(3x^2+1)^{99} \cdot 6x$$

$$\boxed{y'(0) = 0}$$

(b) $y = \dfrac{\sin x}{\sqrt{2 + \sin x}}$

$$\frac{dy}{dx} = \frac{\cos(x) \cdot \sqrt{2+\sin x} - \sin(x) \cdot \frac{1}{2\sqrt{2+\sin x}} \cdot \cos x}{2+\sin x}$$

$$y'(0) = \frac{1 \cdot \sqrt{2} - 0}{2} = \boxed{\frac{\sqrt{2}}{2}}$$

(c) $y^3 + y = x^3 - x$

$$y^2 3\frac{dy}{dx} + \frac{dy}{dx} \cdot 1 = 3x^2 - 1$$

$$(3y^2+1)\frac{dy}{dx} =$$

$$\frac{dy}{dx} = \frac{3x^2-1}{3y^2+1}$$

$$x = 0, \quad y^3 + y = 0$$

$$y(y^2+1) = 0 \Rightarrow y = 0$$

$$y'(0) = \frac{0^2-1}{0^2+1} = \boxed{-1}$$

2. Use a linear approximation to estimate $\sqrt[3]{28}$. (Be sure to identify the function f you are approximating and the point x_0 at which your linear approximation is tangent to f.)

$$f(x) = \sqrt[3]{x} = x^{1/3}$$

$$f(27) = \sqrt[3]{27} = 3$$

$$f'(x) = \frac{1}{3}x^{-2/3}$$

$$f'(27) = \frac{1}{3}(27)^{-2/3} = \frac{1}{3}(3^3)^{-2/3} = \frac{1}{3}(3^{-2}) = \frac{1}{27}$$

$$f(28) \approx f(27) + f'(27)(28-27)$$

$$\approx \sqrt[3]{27} + \frac{1}{27}(1)$$

$$\approx 3 + \frac{1}{27}$$

3. Let $f(x) = 2x^3 + 10x - 1$.

(a) Determine whether f is increasing, decreasing or neither on $\mathbb{R}$. Justify your answer.

$f'(x) = 6x^2 + 10$

$f' > 0$ for all x means

$f \uparrow$ for all $x \in \mathbb{R}$

(b) Explain why $f(x) = 0$ must have exactly one real solution.

$f(1) = 2(1)^3 + 10(1) - 1$

$= 2 + 10 - 1 = 12 - 1 = 11$

$f(-1) = 2(-1)^3 + 10(-1) - 1 = -2 - 10 - 1 = -13$

$f(x) = 0 \Rightarrow x = f^{-1}(0)$. Since f is always increasing, it is invertible and one-to-one.

(c) Use **two** iterations (i.e., x_1 and x_2) of Newton's Method to approximate the solution to $f(x) = 0$. Use $x_0 = 0$ as an initial guess. (You should write down how to find x_1 and x_2, but feel free to use your calculator to compute them.)

$X_0 = 0$

$X_1 = X_0 - \dfrac{f(X_0)}{f'(X_0)} = 0 - \dfrac{f(0)}{f'(0)}$

$= -\dfrac{(-1)}{(10)} = \dfrac{1}{10} = 0.1$

$X_2 = X_1 - \dfrac{f(X_1)}{f'(X_1)} = 0.1 - \dfrac{f(0.1)}{f'(0.1)}$

$= 0.1 - \dfrac{[2(0.1)^3 + 10(0.1) - 1]}{[6(0.1)^2 + 10]}$

$\approx 0.1 - \dfrac{0.002}{10.06} = \dfrac{1.004}{10.06}$

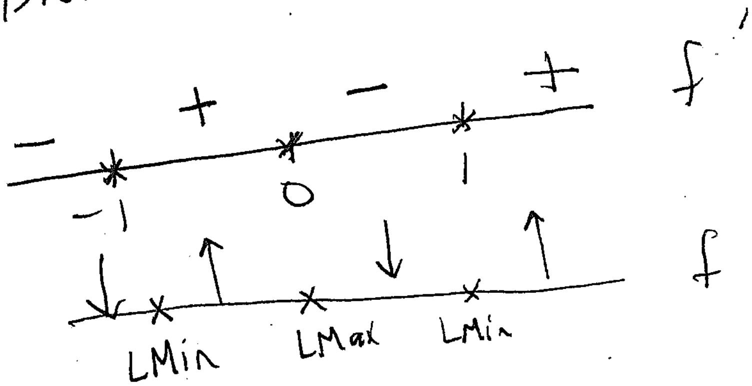
$\approx$

4. Let $f(x) = (x^2 - 1)^{2/3}$. Find all critical numbers for f and determine whether each represents a local maximum, a local minimum or neither.

$$f'(x) = \frac{2}{3}(x^2-1)^{-1/3} \cdot (2x)$$

$$f' = 0 \Rightarrow 2x = 0 \Rightarrow x = 0$$

$$f' \text{ DNE} \Rightarrow x^2 - 1 = 0 \Rightarrow x = 1, -1$$



5. As you blow up a helium balloon, the volume changes at a constant rate of 20 cm^3/sec. How fast is the radius changing when the radius is 5 cm? Assume a spherical balloon with volume $V = \frac{4}{3}\pi r^3$ and think of both V and r as functions of time t.

$$\frac{dV}{dt} = 20 \frac{cm^3}{sec}$$

$$\frac{dr}{dt}\frac{dV}{dr} = \frac{dV}{dt}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dr}} = \frac{20 \, cm^3/sec}{4\pi(5)^2 cm^2} = \frac{20}{100\pi}\frac{cm}{sec}$$

$$= \frac{1}{5\pi} \, cm/sec$$

EXTRA: If possible, use L'Hopital's Rule to evaluate the limit. If L'Hopital's Rule cannot be applied, explain why.

$$\lim_{x\to 0} \frac{e^{e^x} - e}{x} = \frac{\text{"}0\text{"}}{\text{"}0\text{"}}$$

L'H

$$= \lim_{x\to 0} \frac{e^{e^x} \cdot e^x}{1} = e^{e^0} \cdot e^0 = e^1 \cdot 1$$

$$= e$$