MATH 110 Exam 2

October 20, 2004

Name: Key

Please write up your solutions in the space provided. You may use the back of the pages for scratch work and for extra space if you need it. If you wish me to grade any work on the backs of pages, please draw attention to it on the front pages. Boxed answers are appreciated for numerical problems. Although calculators are permitted for this exam, you must provide appropriate justification for your final answers. Please carefully read the instructions for each problem.

There are 4 problems worth 55 points and 1 extra credit problem.

Good luck!

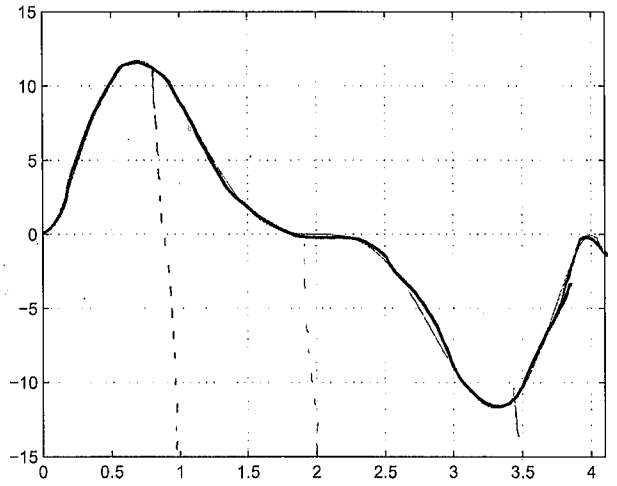
Question	Points	Score
1	10	
2	15	
3	15	
4	15	
EXTRA	3	
Total	55	

1. (a) For the function g whose graph is shown here, arrange the following numbers in increasing order.

$$0 \qquad g'(1) \qquad g'(2) \qquad g'(3.5) \qquad g''(4)$$

$g''(4) < g'(2) < 0 = g'(2) < g'(3.5)$

You can argue which is more negative, $g''(4)$ or $g'(1)$



$g'(1) < 0$ ↓g

$g'(3.5) > 0$ g↑

$g'(2) = 0$

$g'(4) = 0$

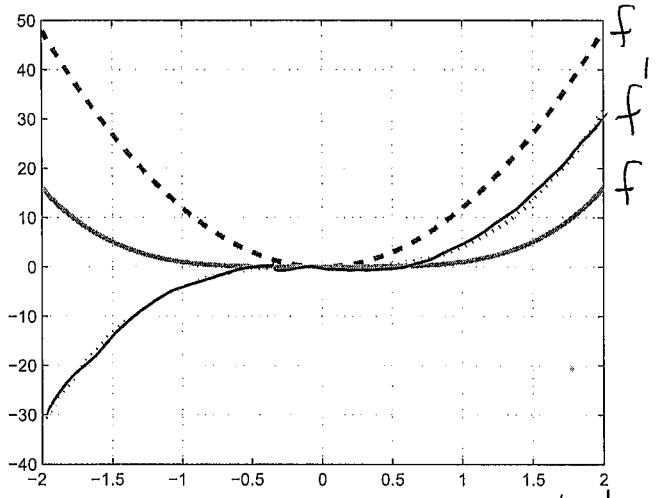
$g''(4) < 0$

since $g'(4^-) > 0$

$g'(4^+) < 0$

g(4)↓

(b) The figure below shows the graphs of f, f', and f''. Identify each curve and explain your choices. (If you need more space for your explanation, continue on the back of this page.)



∴ The solid line must be f' since its derivative must be positive everywhere.

2. Find the derivative of each funtion.

(a) $f(x) = \pi x^2 + \frac{6}{\sqrt[3]{x^4}} + 2^x$

$$f'(x) = 2\pi x + 6\left(-\frac{4}{3}x^{-4/3-1}\right) + 2^x \ln 2$$

$$= 2\pi x - 8x^{-7/3} + 2^x \ln 2$$

Power Rule, Exponential Rule, Constant Multiple, Sum

(b) $g(x) = x^2 \ln x$

$$g'(x) = x^2 \cdot \frac{1}{x} + 2x \cdot \ln x$$

Product Rule

(c) $h(x) = \frac{5e^x}{1+\cos x}$

$$h'(x) = \frac{(5e^x)\cdot(1+\cos x) - (5e^x)(-\sin x)}{(1+\cos x)^2}$$

Quotient Rule

3. A particle moves on a vertical line so that its coordinate at time t is $y = t^3 - 12t + 3$, $t \geq 0$.

(a) What is the average velocity over the interval $[0, 3]$?

$f(t) = t^3 - 12t + 3$

$$\text{Ave vel} = \frac{f(3) - f(0)}{3 - 0} = \frac{3^3 - 12 \cdot 3 + 3 - (3)}{3}$$

$$= \frac{27 - 36 + 3 - 3}{3} = -\frac{9}{3} = \boxed{-3}$$

(b) What is the instantaneous velocity of the particle at time $t = 3$?

$$\text{Inst vel} = f'(3) = 3 \cdot 3^2 - 12 = 27 - 12 = \boxed{15}$$

$f'(t) = 3t^2 - 12$

(c) When is the particle moving downward?

When $f'(t) < 0$, particle moves down

$3t^2 - 12 < 0$

$3t^2 < 12$

$t^2 < 4$

$t < 2$ or $t > -2$

Since $t \geq 0$, the particle moves down when $0 \leq t \leq 2$.

4. Let $f(x) = x^3 - 2x$.

(a) Use the limit definition of the derivative to find $f'(2)$.

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim \frac{x^3 - 2x - [2^3 - 2 \cdot 2]}{x - 2}$$

$$= \lim_{x \to 2} \frac{x^3 - 2x - 4}{x - 2} = \lim_{x \to 2} \frac{x^3 - 8 - 2x - 4 + 8}{x - 2}$$

$$= \lim_{x \to 2} \frac{(x-2)(x^2 + 2x + 4) - 2x + 4}{x - 2} = \lim_{x-2} (x-2)\left[\frac{x^2 + 2x + 4 - 2}{x - 2}\right.$$

$$= \lim_{x \to 2} x^2 + 2x + 2 = 10$$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$\frac{(2+h)^3 - 4}{h} = \frac{8 + 2h \cdot 3 + 3^2 \cdot 2 \cdot h^2 + h^3 - 4}{h} + 2(2+h)$$

$$\frac{6h + 18h^2 + h^3 + 4h}{h} =$$

$$= 10 + 18h + h^2$$

$$f'(2) = \lim_{h \to 0} 10 + 18h + h^2$$

$$= 10$$

(b) Find an equation of the tangent line to the curve $x^3 - 2x$ at the point $(2, 4)$.

$$f'(2) = 10 = m$$

$$(x_1, y_1) = (2, 4)$$

$$y - y_1 = m(x - x_1)$$

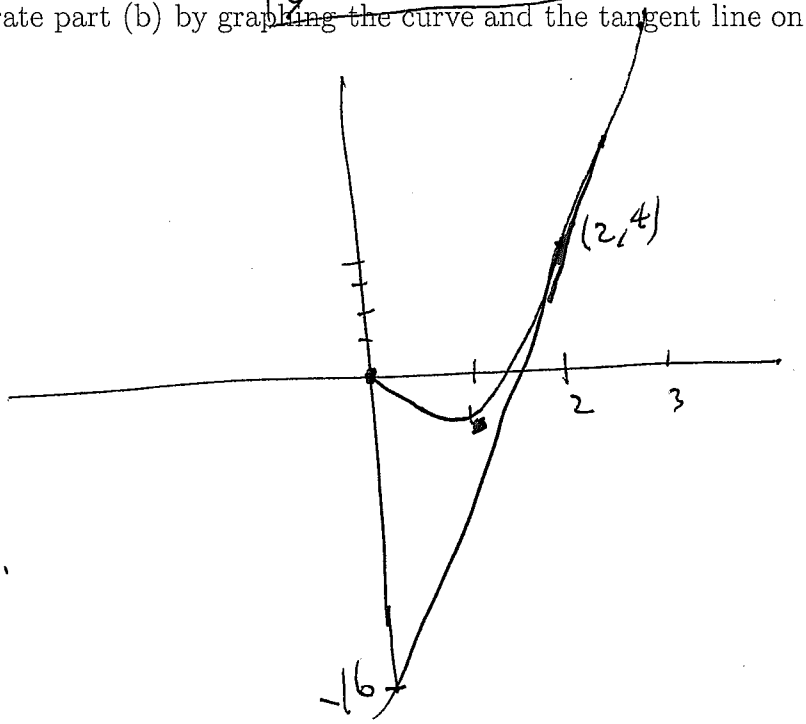
$$y - 4 = 10(x - 2)$$

$$y - 4 = 10x - 20$$

$$y = 10x - 20 + 4$$

$$y = 10x - 16$$

(c) Illustrate part (b) by graphing the curve and the tangent line on the same set of axes.



x	$f(x)$
0	0
1	-1
2	4
3	21

EXTRA: The total cost of repaying a student loan at an interest rate of r percent per year is $C = f(r)$.

(a) What is the meaning of $f'(8)$? What are its units?

$f'(8)$ = The rate of which the cost of the loan changes per percent change in interest rate when the current rate is 8%, its units are $/%

(b) What does the statement $f'(10) = 1200$ mean?

When the rate is 10% it cost $1200 per 1 percent change in the interest rate

(c) Is $f'(r)$ always positive or does it change sign? Explain your answer.

As r increases the cost C(r) increases