REVIEW EXERCISES SOLUTIONS

1. $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

SIMPLIFY DIFF QUOTIENT

$$\frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h \cdot (\sqrt{x+h} + \sqrt{x})} = \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{h}{h\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

TAKE LIMIT

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$$

2. PROVE $(C f(x) + D)' = C f'(x)$

$$\lim_{h \to 0} \frac{C f(x+h) + D - [C f(x) + D]}{h} \overset{\text{Algeb}}{=} \lim_{h \to 0} \frac{C f(x+h) + D - C f(x) - D}{h}$$

$$\overset{\text{Algebra}}{=} \lim_{h \to 0} \frac{C f(x+h) - C f(x)}{h} \overset{\text{Algebra}}{=} \lim_{h \to 0} C \frac{[f(x+h) - f(x)]}{h}$$

$$\overset{\text{Const Mult Rule for Limits}}{=} C \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$[C f(x) + D]' = C f'(x)$$

3. $f(x) = \ln(\cot(x)\,47^x)$

$$f'(x) = \frac{1}{\cot(x)\,47^x}\cdot(\cot(x)\,47^x)'$$

$$= \frac{1}{\cot(x)\,47^x}\left[(\cot(x))'\,47^x + \cot(x)\,(47^x)'\right]$$

$$= \frac{1}{\cot(x)\,47^x}\left[-\csc^2 x\cdot 47^x + \cot(x)\,47^x \ln 47\right]$$

$$(\cot(x))' = \left(\frac{1}{\tan(x)}\right)' = -\frac{\sec^2(x)}{\tan^2(x)} = \frac{-1}{\sin^2(x)} = -\csc^2 x$$

4. $y = t\sin t$

$$y' = \sin t + t\cos t$$

$$y'' = \cos t + \cos t - t\sin t$$

$$y'' = 2\cos t - t\sin t$$

$$y'' = 2\cos t - y$$

Yes $y = t\sin t$ is the sol' to the DE $y'' = 2\cos t - y$.

5. $$\lim_{t\to 0}\frac{\ln(47+t) - \ln(47)}{t} = f'(47) \quad \text{where } f(t) = \ln t$$

$$= \frac{1}{47}$$

6. $$\left(\frac{f(x)}{g(x)}\right)' = \left[f(x)(g(x))^{-1}\right]' = f'(x)(g(x))^{-1} \underset{\text{PRODUCT RULE}}{+} f(x)\underset{\text{CHAIN RULE}}{\left[(g(x))^{-1}\right]'}$$

$$= \frac{f'}{g} + f\left(\frac{-1}{g^2}\cdot g'\right) = \frac{gf'}{g^2} + \frac{-fg'}{g^2} = \frac{f'g - fg'}{g^2}$$

6(b) $g(x)h(x) = f(x)$

Differentiate both sides.

$$g'h + gh' = f'$$

$$h = \frac{f}{g}$$

$$g'\frac{f}{g} + gh' = f'$$

$$gh' = f' - g'\frac{f}{g}$$

$$h' = \frac{f'}{g} - g'\frac{f}{g^2}$$

$$= \frac{f'g}{g^2} - \frac{g'f}{g^2} = \frac{f'g - g'f}{g^2}$$

QUOTIENT RULE

7.

$$\lim_{x \to a} f(x) = \lim_{x \to a} \left[f(x) - f(a) + f(a) \right]$$

$$= \lim_{x \to a} \left[\frac{f(x) - f(a)}{(x - a)} (x - a) + f(a) \right]$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \lim_{x \to a} (x - a) + \lim_{x \to a} f(a)$$

$$= f'(a) \cdot 0 + f(a)$$

$$\lim_{x \to a} f(x) = f(a)$$

8. $f(x) = \begin{cases} \dfrac{x^2-2x}{x-2}, & \text{if } x \neq 2 \\ \lim\limits_{x \to 1} \dfrac{Kx-K}{2x-2} & \text{if } x = 2 \end{cases}$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x^2-2x}{x-2} = \lim_{x \to 2} \frac{x\cancel{(x-2)}}{\cancel{x-2}} = \lim_{x \to 2} x = 2$$

$$\lim_{x \to 1} \frac{K \cdot x - K}{2x-2} = \lim_{x \to 1} \frac{K(x-1)}{2(x-1)} = \frac{K}{2} \overset{?}{=} 2 \Rightarrow K = 4$$

~~There is NO K value that will make f(x) continuous at x = 2~~

9. $f(x)$ is the number of snarfs living in Snarfville, x years from now.

Units of $f'(x)$ is in $\dfrac{\text{snarfs}}{\text{year}}$

Units of $f''(x)$ is in $\dfrac{\text{snarfs}}{\text{year}^2}$

10. $f(x) = \dfrac{1}{x}$ $\quad x_0 = 2 \quad f(x_0) = \dfrac{1}{x_0} = \dfrac{1}{2}$

$f'(x) = \dfrac{-1}{x^2} \quad f'(x_0) = \dfrac{-1}{x_0^2} = -\dfrac{1}{2^2} = \dfrac{-1}{4}$

~~f(1.7)~~

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$f(x) \approx \frac{1}{2} - \frac{1}{4}(x-2)$$

$$f(1.7) = \frac{1}{1.7} \approx \frac{1}{2} - \frac{1}{4}(1.7-2)$$

$$\approx \frac{1}{2} - \frac{1}{4}(-0.3)$$

$\dfrac{1}{1.7} = 0.588235 \approx 0.575$
exactly

$\approx 0.5 + 0.25(0.3) \approx 0.5 + 0.075$
$= 0.575$