Calc 110 AMP Facilitator: Eric Fox
AMP WS#8: Mock Exam

Note: Even though this is a mock exam, feel free to work with others and ask me any questions you have. I will not be doing homework questions until the end of the workshop because I want to focus on helping people with the mock exam. Good luck – don't be discouraged, this is supposed to be challenging.

1. Evaluate $f'(x)$ (you don't need to simplify!)

a) $f(x) = 10(x^2+3)^4$

$$f'(x) = 10 \cdot 4(x^2+3)^3 \cdot 2x$$

Chain Rule, Power Rule

b) $f(x) = \ln\left(\sqrt{x^3+42x}\right)$

$$f'(x) = \frac{1}{\sqrt{x^3+42x}} \left(\sqrt{x^3+42x}\right)' = \frac{1}{\sqrt{x^3+42x}} \cdot \frac{1}{2\sqrt{x^3+42x}} \cdot (3x^2+42)$$

Chain Rule

c) $f(x) = b^{ax+c}$ where a, b, c are real numbers

$$f'(x) = b^{ax+c} \cdot \ln(b) \cdot (ax+c)'$$
$$= b^{ax+c} \cdot \ln(b) \cdot a$$

d) Given $g(x) = \tan x$ and $h(x) = xe^{2x}$, evaluate the derivative of $f(x) = g(h(x))$

$$f'(x) = g'(h(x))\, h'(x) = \sec^2(xe^{2x}) \cdot \left[e^{2x} + 2xe^{2x}\right]$$

$$g'(x) = \sec^2(x)$$
$$h'(x) = 1 \cdot e^{2x} + x \cdot 2e^{2x}$$

2. Use the *quotient rule* to prove that $\frac{d[\cot x]}{dx} = -\csc^2(x)$

$$\frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{\frac{d}{dx}(1)\cdot \tan x - \frac{d}{dx}(\tan x)\cdot 1}{\tan^2 x}$$

$$= -\frac{\sec^2(x)}{\tan^2(x)}$$

$$= -\frac{\frac{1}{\cos^2(x)}}{\frac{\sin^2(x)}{\cos^2(x)}} = -\frac{\cos^2(x)}{\cos^2(x)}\cdot\frac{1}{\sin^2(x)}$$

$$= -\csc^2(x)$$

3. Consider the hyperbolic function $\cosh x = \frac{e^x + e^{-x}}{2}$ and $\sinh x = \frac{e^x - e^{-x}}{2}$. Show that:

a) $\frac{d[\cosh x]}{dx} = \sinh x$

$$\frac{d(\cosh x)}{dx} = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{2}\right) = \frac{1}{2}\frac{d}{dx}(e^x) + \frac{1}{2}\frac{d}{dx}(e^{-x})$$

$$= \frac{1}{2}e^x - \frac{1}{2}e^{-x} = \frac{e^x - e^{-x}}{2}$$

$$= \sinh(x)$$

b) $\frac{d[\sinh x]}{dx} = \cosh x$

$$\frac{d}{dx}(\sinh x) = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) = \frac{1}{2}\frac{d}{dx}(e^x) - \frac{1}{2}\frac{d}{dx}(e^{-x})$$

$$= \frac{1}{2}e^x - \frac{1}{2}(-e^{-x}) = \frac{1}{2}e^x + \frac{1}{2}e^{-x} = \cosh(x)$$

c) both $\cosh x$ and $\sinh x$ are solutions to the differential equation $y'' - y = 0$ (hint: just plug in the functions into the equation)

$\frac{d}{\cancel{dx}}$

$y = \sinh x$
$y' = \cosh x$
$y'' = \sinh x$
$y'' - y = \sinh x - \sinh x$
$= 0$

$y = \cosh(x)$
$y' = \sinh(x)$
$y'' = \cosh(x)$
$y'' - y = \cosh x - \cosh x$
$= 0$ ✓

4. $\lim_{x\to 0}\frac{\ln(1+x)}{x}$ (Hint: This is the limit definition of a derivative of a function at a certain point. Figure out that function and evaluate the derivative at that point.)

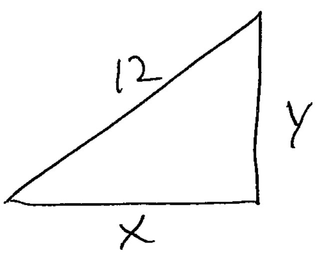
$$\lim_{x\to 0}\frac{\ln(1+x)}{x} = \lim_{x\to 0}\frac{\ln(1+x)-\ln(1)}{x} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$= f'(1) \quad \text{where } f(x)=\ln(x),\ a=1$$

$$= \left.\frac{1}{x}\right|_{x=1}$$

$$= 1$$

5. A snarf decides to climb up a 12 foot ladder leaning against a plumb tree. When the base of the ladder is 3ft from the tree it is sliding away from the tree at a rate of .1 ft/sec. How fast is the top of the ladder falling at that moment?



$$x^2+y^2=12$$

$$\frac{dx}{dt} = -0.1\ \frac{ft}{sec}$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

When $x=3$, $y=\sqrt{12^2-3^2}=\sqrt{144-9}=\sqrt{135}$

$$2\cdot 3(-0.1)+2\cdot\sqrt{135}\,\frac{dy}{dt}=0$$

$$\frac{dy}{dt} = -\frac{0.3}{\sqrt{135}}\ ft/sec$$

6. Use the limit definition of the derivative to find the derivative of $f(x)=\dfrac{1}{48x^2}$. Then find the equation for the tangent line a $x=-1/2$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{48(x+h)^2}-\frac{1}{48x^2}}{h}$$

$$= \frac{1}{48}\lim_{h\to 0}\frac{\frac{x^2-(x+h)^2}{(x+h)^2\,x^2}}{h} = \frac{1}{48}\lim_{h\to 0}\frac{x^2-[x^2+2hx+h^2]}{x^2h\cdot(x+h)^2}$$

$$= \frac{1}{48}\lim_{h\to 0}\frac{-2hx-h^2}{x^2\cdot h\cdot(x+h)^2} = \frac{1}{48}\lim_{h\to 0}\frac{-2x-h}{x^2\cdot(x+h)^2}$$

$$= \frac{1}{48}\frac{(-1)2x}{x^2\cdot x^2} = -\frac{1}{48}\frac{2}{x^3} = \frac{-2}{48x^3} = f'(x)$$

$$-24x^{-3} = \frac{-1}{24x^3} =$$

At $x=-1/2$, $f'(-1/2) = -\dfrac{1}{24(-\frac{1}{2})^3} = \dfrac{-1}{24(-\frac{1}{8})} = \dfrac{-1}{-3} = \dfrac{1}{3}$

$f(-1/2) = \dfrac{1}{48(-\frac{1}{2})^2} = \dfrac{1}{48\cdot\frac{1}{4}} = \dfrac{1}{12}$ $\qquad m = \dfrac{1}{3}$

$(x_1, y_1) = (-1/2, 1/12)$

$$y - \frac{1}{12} = \frac{1}{3}\left(x - -\frac{1}{2}\right)$$

$$y = \frac{1}{3}\left(x+\frac{1}{2}\right)+\frac{1}{12}$$

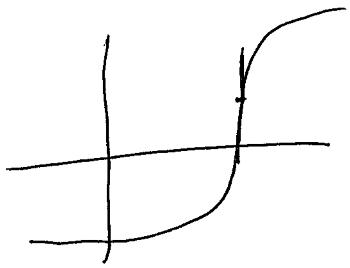
$$= \frac{1}{3}x + \frac{1}{6} + \frac{1}{12} = \frac{1}{3}x + \frac{3}{12} = \frac{1}{3}x + \frac{1}{4}$$

7. T/F. If you put that the answer is false give and explanation as to why.

F $f(x) = |x|$ is differentiable everywhere.

$|x|$ is not locally linear at $x = 0$ and thus $f'(0)$ does not exist

T A function is not differentiable at points of vertical tangency.



The slope of a vertical line does not exist = slope of function at point of vertical tangency

F If a function is continuous at a point x_0, then that function is differentiable at x_0.

Diff $\Rightarrow$ Cont

Cont does NOT imply Differentiability!!

$f(x) = |x|$ is cont at $x_0 = 0$, $f'(x_0)$ DNE

T If a function is differentiable over the open interval (a,b), where a and b are real numbers, then the two sided limit $f'(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$ exists for any x_0 inside of that interval.

Definition of derivative!