Calc 110 AMP Facilitator: Eric Fox
AMP WS#8: Mock Exam

Note: Even though this is a mock exam, feel free to work with others and ask me any questions you have. I will not be doing homework questions until the end of the workshop because I want to focus on helping people with the mock exam. Good luck—don’t be discouraged, this is supposed to be challenging.

1. Evaluate \( f'(x) \) (you don’t need to simplify!)
   a) \( f(x) = 10(x^2 + 3)^4 \)
   \[
   f'(x) = 10 \cdot 4(x^2 + 3)^3 \cdot 2x \quad \text{Chain Rule, Power Rule}
   \]

   b) \( f(x) = \ln(\sqrt{x^2 + 42x}) \)
   \[
   f'(x) = \frac{1}{\sqrt{x^2 + 42x}} \cdot \frac{1}{2 \sqrt{x^2 + 42x}} \cdot (3x^2 + 42) \quad \text{Chain Rule}
   \]

   c) \( f(x) = e^{ax+c} \) where \( a, b, c \) are real numbers
   \[
   f'(x) = b^{ax+c} \cdot ln(b) \cdot (a x + c)'
   = b^{ax+c} \cdot ln(b) \cdot a
   \]

   d) Given \( g(x) = \tan x \) and \( h(x) = xe^{2x} \), evaluate the derivative of \( f(x) = g(h(x)) \)
   \[
   f'(x) = g'(h(x)) \cdot h'(x)
   = \sec^2(xe^{2x}) \cdot [e^{2x} + 2xe^{2x}]
   \]
   \[
   g'(x) = \sec^2(x)
   h'(x) = e^{2x} + x \cdot 2e^{2x}
   \]
2. Use the quotient rule to prove that \( \frac{d[\cot x]}{dx} = -\csc^2(x) \)

\[
\frac{d}{dx} \left( \frac{1}{\tan x} \right) = \frac{d(1)}{dx} \cdot \tan x - \frac{d(\tan x)}{dx} \cdot 1 + \tan^2 x
\]

\[
= -\sec^2(x)
\]

\[
= -\frac{1}{\cos^2(x)}
\]

\[
= -\frac{\cos^2(x)}{\sin^2(x)} \cdot \frac{1}{\cos^2(x)}
\]

\[
= -\csc^2(x)
\]

3. Consider the hyperbolic function \( \cosh x = \frac{e^x + e^{-x}}{2} \) and \( \sinh x = \frac{e^x - e^{-x}}{2} \). Show that:

a) \( \frac{d[\cosh x]}{dx} = \sinh x \)

\[
\frac{d[\cosh x]}{dx} = \frac{d}{dx} \left( \frac{e^x + e^{-x}}{2} \right) = \frac{1}{2} \frac{d}{dx}(e^x) + \frac{1}{2} \frac{d}{dx}(e^{-x})
\]

\[
= \frac{1}{2} e^x - \frac{1}{2} e^{-x} = \frac{e^x - e^{-x}}{2}
\]

\[
= \sinh x
\]

b) \( \frac{d[\sinh x]}{dx} = \cosh x \)

\[
\frac{d[\sinh x]}{dx} = \frac{d}{dx} \left( \frac{e^x - e^{-x}}{2} \right) = \frac{1}{2} \frac{d}{dx}(e^x) - \frac{1}{2} \frac{d}{dx}(e^{-x})
\]

\[
= \frac{1}{2} e^x - \frac{1}{2} (-e^{-x}) = \frac{e^x + e^{-x}}{2} = \cosh x
\]

c) both \( \cosh x \) and \( \sinh x \) are solutions to the differential equation \( y'' - y = 0 \) (hint: just plug in the functions into the equation)

\[
\frac{d}{dx} \sinh x = \cosh x \]

\[
\frac{d}{dx} \cosh x = \sinh x
\]

\[
\frac{d}{dx} \sinh x = \cosh x
\]

\[
\frac{d}{dx} \cosh x = \sinh x
\]

\[
\sinh x - \sinh x = 0
\]

\[
\cosh x - \cosh x = 0
\]

\[
\cosh x - \cosh x = 0
\]
4. \( \lim_{x \to 0} \frac{\ln(1+x)}{x} \) (Hint: This is the limit definition of a derivative of a function at a certain point. Figure out that function and evaluate the derivative at that point.)

\[
\lim_{x \to 0} \frac{\ln(1+x)}{x} = \lim_{h \to 0} \frac{\ln(1+h) - \ln(1)}{h} = f'(1) \\
= f'(1) \quad \text{where} \quad f(x) = \ln(x) \\
= \frac{1}{x} \bigg|_{x=1} \\
= 1
\]

5. A snarf decides to climb up a 12 foot ladder leaning against a plumb tree. When the base of the ladder is 3ft from the tree it is sliding away from the tree at a rate of .1 ft/sec. How fast is the top of the ladder falling at that moment?

\[x^2 + y^2 = 12\]

\[
\frac{dx}{dt} = -0.1 \text{ ft/sec}
\]

\[
2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0
\]

When \( x = 3 \), \( y = \sqrt{12^2 - 3^2} = \sqrt{144 - 9} = \sqrt{135} \)

\[
2 \cdot 3 \left(-0.1\right) + 2 \cdot \sqrt{135} \frac{dy}{dt} = 0
\]

\[
\frac{dy}{dt} = -0.3 \frac{\text{ft/sec}}{\sqrt{135}}
\]
6. Use the limit definition of the derivative to find the derivative of \( f(x) = \frac{1}{48x^2} \). Then find the equation for the tangent line at \( x = -1/2 \).

\[
\begin{align*}
\frac{f'(x)}{h} &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{1}{48} \left( \frac{x^2 - (x+h)^2}{(x+h)^2 \cdot x^2} \right) \\
&= \lim_{h \to 0} \frac{1}{48} \left( \frac{-2hx - h^2}{x^2 \cdot (x+h)^2} \right) \\
&= \lim_{h \to 0} \frac{1}{48} \left( \frac{-2x - h}{x^2 \cdot (x+h)^2} \right) \\
&= \lim_{h \to 0} \frac{-2x - h}{48h} \\
&= \lim_{h \to 0} \frac{-2x}{48} \\
&= \frac{1}{24x^3} \\
\end{align*}
\]

At \( x = -\frac{1}{2} \), \( f'(-\frac{1}{2}) = -\frac{1}{24 \left( \frac{1}{2} \right)^3} = -\frac{1}{24 \left( \frac{1}{8} \right)} = -\frac{1}{\frac{3}{3}} = \frac{-1}{3} \)

\[
\begin{align*}
f(-\frac{1}{2}) &= \frac{1}{48 \left( -\frac{1}{2} \right)^2} = \frac{1}{48 \cdot \frac{1}{4}} = \frac{1}{12} \\
M &= \frac{1}{3} \left( x, y \right) = \left( -\frac{1}{2}, \frac{1}{12} \right) \\
\end{align*}
\]

\[
\begin{align*}
Y &= \frac{1}{3} \left( x + \frac{1}{2} \right) + \frac{1}{12} \\
&= \frac{1}{3} \left( x + \frac{1}{2} \right) + \frac{1}{12} \\
&= \frac{1}{3} \left( x + \frac{1}{6} + \frac{1}{12} \right) \\
&= \frac{1}{3} \left( x + \frac{3}{12} + \frac{1}{12} \right) \\
&= \frac{1}{3} \left( x + \frac{4}{12} \right) \\
&= \frac{1}{3} \left( x + \frac{1}{3} \right) \\
\end{align*}
\]
7. T/F. If you put that the answer is false give and explanation as to why.
F \( f(x) = |x| \) is differentiable everywhere.

\( |x| \) is not locally linear at \( x = 0 \)
and thus \( f'(0) \) does not exist

T A function is not differentiable at points of vertical tangency.

\[
\begin{align*}
The \text{ slope of a} \\
\text{vertical line} \\
\text{does not exist} \\
= \text{slope of function} \\
\text{at point of vertical tangency}
\end{align*}
\]

F If a function is continuous at a point \( x_0 \), then that function is differentiable at \( x_0 \).

\[
\text{Diff} \Rightarrow \text{Cont} \\
\text{Cont does not imply differentiability!!}
\]

\[
f'(x) = \{ x \} \text{ is cont at } x_0 = 0; \ f'(x_0) \ \text{DNE}
\]

F If a function is differentiable over the open interval (a,b), where a and b are real numbers, then the two sided limit \( f'(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \) exists for any \( x_0 \) inside of that interval.

Definition of derivative.