Occidental College Department of Mathematics
Gateway – Exponents
Help Sheet

The Gateway room, Fowler 301, is staffed Monday through Thursday, 12:30-1:30, for retakes and tutoring. If you can't make it any of those times, contact Professor Lawrence for an appointment (Fowler 324, x2647, DonLoxy.edu).

1. The key rule: $x^a x^b = x^{a+b}$. This holds for ALL a and b! Recall $x^0 = 1$. Ex: $x^2 x^{-5} =$

2. The key rule: $(x^a)^b = x^{ab}$. Here you multiply the exponents. Ex: $(x^2)^{-5} =$

3. The problems here are just a combination of 1 and 2 above. As with all other expressions, work from the inside out, simplifying as you go. Ex: $(xx^{-5}x^3)^{-2} =$

4. This is just a combination of 1 and 2 again, except more variables are introduced. Remember that you can combine x's with other x's and y's with other y's (e.g. $x^2y \cdot x^5y^3 = x^{2+5}y^{1+3} = x^7y^4$). But you wouldn't combine x's with y's. Ex: $(2x^2y)(-xy^2) =$

5. Key rule: $x^{a/b} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$. Ex: $(27)^{2/3} =$

6. You're going to use the same rule as 5, just in reverse. So $\sqrt{x^4} = x^{4/2} = x^2$. Ex: Rewrite $\sqrt[3]{x^6y^3z^7}$ without roots (just use exponents).

7. First, to simplify a fractional expression with exponents, you need to remember the rule: $\frac{x^a}{x^b} = x^{a-b}$. (Note the relationship with the multiplication rule in 1!) Then, to get rid of negative exponents, just remember that $x^{-a} = \frac{1}{x^a}$. So if you have a negative exponent in the numerator (top), just take it to the denominator (bottom) with a positive exponent. The reverse works too: If you have a negative exponent in the denominator, just take it to the numerator with a positive exponent. Ex: Leave no negative exponent in $\frac{xyz^{-2}}{x^{-2}yz^2}$

8. Here, the whole expression is taken to the negative exponent. But remember that the rules in 1 and 2 hold for negative exponents too! Take the exponent first; then simplify. Ex: Leave no negative exponent in $\left(\frac{xy^{-2}}{xy^2}\right)^{-1}$

9. You can solve an equation like $2^{4x} = 8$ using logarithms, but you don't have to use them. You can simply rewrite the right hand side using the same base, 2^3. Since you have two quantities with the same base equal to each other, you can set the two exponents equal to each other, $4x = 3$, and solve for x.

10. Your approach to a more complicated equation with exponents is to first simplify it using addition/subtraction and multiplication/division until you get it in the above form. Another helpful hint: $4^x = (2^2)^x = 2^{2x}$. Ex: Solve $8 + 4^{x+1} = 10$.