

## GATEWAY – EQUATIONS AND INEQUALITIES HELP SHEET

The Gateway Room, Fowler 301, is staffed for tutoring and retakes Monday through Thursday, 12:30-1:30.

(1) and (2) Solve a linear equation for  $x$ . If there are parentheses, remove them by distributing. Move all the  $x$ 's to the left, and all the constants to the right. Then divide by the coefficient of  $x$ .

Examples:

$$\begin{array}{ll} 2x - 3 = 4 - x & 3(x - a) = b \\ 3x = 7 & 3x - 3a = b \\ x = \frac{7}{3} & 3x = b + 3a \\ & x = \frac{b + 3a}{3} \end{array}$$

Exercises:  $2(3 - x) = 3x + 4$        $a\left(\frac{x}{2} - 2\right) = z$

(3) Solve a quadratic equation by factoring.

Example:

$$\begin{array}{l} 2x^2 - 3x - 2 = 0 \\ (2x + 1)(x - 2) = 0 \\ 2x + 1 = 0 \text{ or } x - 2 = 0 \\ x = -\frac{1}{2} \text{ or } x = 2 \end{array}$$

Exercises:  $x^2 - 5x + 6 = 0$        $4x^2 - 4x - 3 = 0$

(4) In this problem, the point is that the only way a fraction can be zero is if the numerator is zero.

Example:

$$\begin{array}{l} \frac{(2x + 1)(x - 3)}{x - 47} = 0 \\ (2x + 1)(x - 3) = 0 \\ 2x + 1 = 0 \text{ or } x - 3 = 0 \\ x = -\frac{1}{2} \text{ or } x = 3 \end{array}$$

Exercises:  $\frac{x + 1}{(x - 4)(2x - 3)} = 0$        $\frac{x(x + 1)(2x - 1)}{x^2 - 2} = 0$

(5) Same as (4), except that the square root in the denominator might require you to reject some solutions.

Example:

$$\frac{(x+2)(x-3)}{\sqrt{x+1}} = 0$$

$$(x+2)(x-3) = 0$$

$$x+2 = 0 \text{ or } x-3 = 0$$

$$x = -2, 3$$

But you can't plug  $x = -2$  back into the  $\sqrt{x+1}$  in the original equation, so reject it; your final answer is  $x = 3$ .

Exercises:  $\frac{(2x-3)(x+4)}{\sqrt{1-x}} = 0$        $\frac{x(x+2)(x+4)}{\sqrt{x-1}} = 0$

(6) and (7) Factor out the common factor, then solve. In number (7), factor out the "most negative" power of  $x$ , so that you have only positive exponents on the  $x$ 's in the rest of the equation.

Examples:

$$2x^3(x+1) + x^3(2x-3) = 0$$

$$x^3[2(x+1) + (2x-3)] = 0$$

$$x^3(4x-1) = 0$$

$$x^3 = 0 \text{ or } 4x-1 = 0$$

$$x = 0, \frac{1}{4}$$

$$x^{-2}(x+4) + x^{-3}(2x+5) = 0$$

$$x^{-3}[x(x+4) + (2x+5)] = 0$$

$$x^{-3}[x^2 + 6x + 5] = 0$$

$$\frac{(x+5)(x+1)}{x^3} = 0$$

$$x = -5, -1 \text{ (as in problem 4)}$$

Exercises:  $(x+2)^2(x+3) + 2(x+2)^2(x-1) = 0$        $2x^{-4}(x-3) + x^{-3}(x+3) = 0$

(8), (9), and (10) Solve the linear inequality, and write your answer in interval notation. Remember that when you multiply or divide by a negative number, you must change the direction of the inequality (or inequalities).

Example:

$$3 \leq 4 - 2x < 10$$

$$-1 \leq -2x < 6$$

$$\frac{1}{2} \geq x > -3$$

$$\text{solution : } (-3, \frac{1}{2}]$$

Exercises:  $5 + 2x < 6$        $0 \geq 3 - 2x > -3$