Applications of Derivatives: Finding Maxima and Minima

Concavity

The second derivative $f''(x)$ gives us information on the **concavity** of the graph of $f(x)$. We say a graph is **concave up** if the graph stays above the tangent line, and it is **concave down** if the graph stays below the tangent line.

If $f'' > 0$ on an interval then the graph of $f(x)$ is *concave up* on that interval.

If $f'' < 0$ on an interval then the graph of $f(x)$ is *concave down* on that interval.

CONCAVE UP **CONCAVE DOWN**

Definition A point at which the function changes concavity is called an *inflection* point. *To find an inflection point we set $f''(x) = 0$ or check where f'' Does Not Exist (D.N.E.).*

To find a possible inflection point we check where $f''(x) = 0$ or where $f''(x)$ does not exist.

Second Derivative Test for local maxima and minima

Let $(c, f(c))$ be a critical point.

If $f''(c) > 0$ then $(c, f(c))$ is a local minimum.

If $f''(c) < 0$ then $(c, f(c))$ is a local maximum.

If $f''(c) = 0$ then $(c, f(c))$ is a possible inflection point.

Examples

[**1**]. Sketch $f(x) = \frac{1}{6}x^3 - 2x$ by determining all local maxima and local minima and inflection points.

[**2**]. Sketch $f(x) = x\ln(x)$ on the interval $0 < x < 1$.

OPTIMIZATION

We will deal with some real life problems where it is important to find the maximum and minimum value of some quantity. So in some sense we are trying to optimize the value of some quantity. All the techniques for finding such values make up the field called *optimization*. We shall be able to apply our experience with modelling together with our knowledge of the First Derivative and Second Derivatives Tests to help us in our optimization efforts.

Examples

[**1**]**.** Show that the rectangle of fixed perimeter P whose area is a maximum is a square.

[**2**]**.** A roman window is shaped like a rectangle surrounded by a semi-circle. If the perimeter is L feet, what are the dimensions of the window of maximum area?