Average and Instantaneous Velocity

Suppose from 8:00 AM to 9:00 AM you travelled 70 miles in the same (positive) direction. What was your average velocity for this trip? ____________

1. What was your exact velocity at 8:10 AM? __________________________

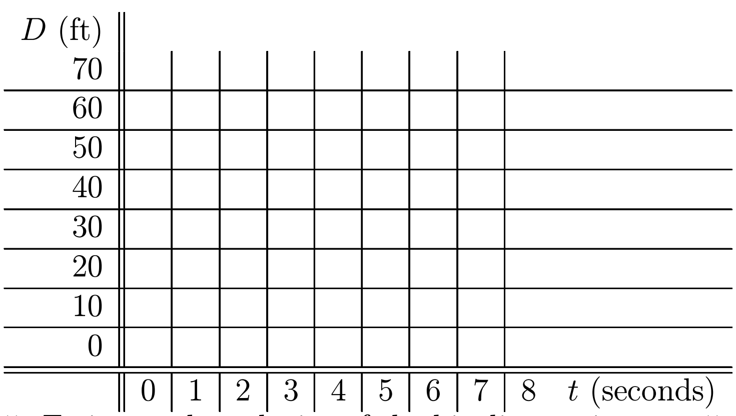
2. Suppose you knew that from 8:09 to 8:15 you travelled 4 miles. What would you guess your exact velocity was at 8:10 AM? _________

3. Suppose the data in the following table had been recorded:

Time	Distance
8:00	0
8:09:00	7
8:09:58	8
8:10:00	8.03
8:15	11
9:00	70

4. Using this data, what would you estimate the exact velocity to be at 8:10:00? _________

Suppose the graph of the distance travelled by a bicylist as a function of time looks as follows.



5. Estimate the velocity of the bicylist at time $t = 5$

6. Estimate the slope of the graph at time $t = 5$

7. During which time intervals is the velocity constant?

8. During which time intervals is the velocity increasing?

9. During which time intervals is the velocity decreasing?

Average and Instantaneous Velocity
We have just had practice estimating velocity given a table (and a graph) of distance travelled versus time elapsed. Suppose the position of a car at time t seconds is given by (the function) $s(t)$ feet.

Then the **average velocity** of the car between $t = a$ and $t = b$ is given by,

$$v_{ave} = \frac{s(b) - s(a)}{b - a}.$$

If we consider the average velocities over successive intervals, each one with b closer to a, we may find that these average velocities approach a definite value v. If so, we call this value v the **instantaneous velocity** at $t = a$ and write

$$v = \lim_{b \to a} \frac{s(b) - s(a)}{b - a}.$$

Sometimes other notations are used. For example, if we let $\Delta t = b - a$, then Δt approaches 0 as b approaches a and we can rewrite the definition of instantaneous velocity as

$$v = \lim_{\Delta t \to 0} \frac{s(a + \Delta t) - s(a)}{\Delta t}.$$

Finding the rate of change of a linear function.
In Unit 1, the rate of change of a function could be calculated using a rate equation. The function itself, however, was unknown. Euler's Method was used to approximate the graph of this unknown function.

In this unit of the course, we will turn this around. We will begin with a known function and try to determine its rate of change. The work we have just done to define average and instantaneous velocity will be the key.

The *average rate of change of a function $y = f(x)$ on an interval* $[a, b]$ is given by the change in the output divided by the change in the input:

$$\frac{\Delta y}{\Delta x} = \frac{\text{change in output}}{\text{change in input}} = \frac{f(b) - f(a)}{b - a}.$$

Example:
What is the average rate of change of the function $f(x) = 3x + 2$ on the interval $[4, 10]$?

What is the instantaneous rate of change of the function $f(x) = 3x + 2$ at the point $x = 4$?

For a **linear function**, the *instantaneous rate of change* has the same value as the *average rate of change*; both are equal to the slope of the line which is its graph.