
Warm-Up

Discuss the following questions:

I. If you say that a variable V is (directly) proportional to a variable X, you mean that:

a) as X increases, V increases.;

b) there is a non-zero constant k (not necessarily negative) such that $V = k \cdot X$;

c) there is a non-zero constant k (not necessarily negative) such that $\Delta V = k \cdot \Delta X$;

II. In using Euler's Method to create a piecewise linear approximation to the solution of the initial value problem

$$H'(t) = -.008 \cdot (H(t) - 3.5), \qquad H(0) = 84.98,$$

why do we estimate ΔH using the formula $\Delta H \approx H'(t)\Delta t$?

Slope Fields

DEFINTION: A **slope field** for a rate equation of the form

$$y'(t) = F(t, y(t))$$

consists of a set of (t, y) coordinate axes, with, at regularly spaced points in the coordinate plane, little sloped line segments. The slope of the line segment centered on a point with coordinates (t, y) has the numerical value $F(t, y(t))$.

A slope field for a rate equation is a useful way to visualize the information provided by the rate equation.

Example

1. Consider the rate equation $y'(t) = \frac{1}{4}t$. Complete the table below, then use it to sketch a slope field for this rate equation:

t:	0	0	0	1	1	1	2	2	2	3	3	3	4	4	4
y:	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
$y'(t)$:															

2. Match the slope fields below with one of the two following rate equations. Explain your choice. (See *H-H*, Section 10.2, p. 495.)

A: $y'(x) = 2x$

B: $y'(x) = y$

Slope Fields and Euler's Method

A slope field for a rate equation can help your visualize solutions to the rate equation. It can also help you visualize how Euler's Method is approximating solutions to the rate equation.

Example

Consider the initial value problem: $y'(t) = \frac{1}{4}t, \quad y(0) = 0.$

3. In the next unit of this course, you will learn techniques you can use to verify that the solution to this initial value problem has the formula $y(t) = \frac{1}{8}t^2$. Complete the following table, then plot a graph of this function on the slope field you constructed in 1. above.

t:	0	1	2	3	4
$y(t)$:					

4. Using Euler's Method, complete the following table to find a piecewise linear function approximating the solution of this same initial value problem, $y'(t) = \frac{1}{4}t, \quad y(0) = 0$, using a stepsize of $\Delta t = 2$. Plot this approximation on the same slope field in 1. above.

t	$y(t)$	Δt	$y'(t)$	$\Delta y \approx y'(t) * \Delta t$
0	0	2		
2		2		
4		2		

5. Using Euler's Method, complete the following table to find a piecewise linear function approximating the solution of this same initial value problem, $y'(t) = \frac{1}{4}t, \quad y(0) = 0$, using a stepsize of $\Delta t = 1$. Plot this approximation on the same slope field in 1. above.

t	$y(t)$	Δt	$y'(t)$	$\Delta y \approx y'(t) * \Delta t$
0	0	1		
1		1		
2		1		
3		1		
4		1		