Preparing for Class 30

Reading: Read Section 5.1

Problems: Complete the handout on Newton's Method.

Monday, November 6

Class 30:

Introduction to Optimization

The work we have done to identify and classify extrema will be applied to optimize functions arising in mathematics and applications.

Preparing for Class 31

Reading: Review *H-H*, Section 5.3; Read *H-H*, Sections 5.2 and 5.5

Problems: *H-H* Section 5.3, # 9, 10, 11, 13, 19, 22; Section 5.2, # 3, 7

Homework Due: Only problems assigned to prepare for Classes 28, 29, and 30 are due at the start of Class 31.

Wednesday, November 8

Class 31:

Modelling Optimization Problems

Optimization problems often arise in mathematics and its applications. It is not often, however, that someone hands you a formula for the function that needs to be optimized. Often some modelling is required to construct this function. This class (and this week's lab) will illustrate this by working through a number of examples. The tools we have developed to find maxima, minima and roots will be useful here once the function to be optimized is constructed.

Take-Home Quiz on Finding Roots, including Newton's Method

Lab: Optimization Problems

Preparing for Class 32

Reading: Review *H-H*, Section 5.5; read *H-H*, Section 10.5.

Problems: Section 5.5, # 4, 5, 6, 8, 9, 11

Friday, November 9

Class 32:

Equilibrium and Inflection Values

Recall that in the first unit of this course, we studied rate equations of the form $y'(t) = F(y(t))$. The work we have done to understand roots, inflection points and extrema can be used to better understand the behavior of solutions of such a rate equation. In particular, we will see how properties of the slope function $F(y)$ determine properties of the solutions $y(t)$.

If there is a value y^* for which $F(y^*) = 0$, then $y'(t) = 0$ when $y(t) = y^*$. Since the value of $y'(t)$ in the rate equation depends explicitly only on the value of y, the fact that $y'(t) = 0$ implies that $y(t)$ will remain at the constant value y^* for all values of t. This value y^* is said to be an *equilibrium value* for the rate equation $y'(t) = F(y(t))$. The behavior of the slope function $F(y)$ for values of y near y^* will determine how solutions of the rate equation passing through a y-value *near* the equilibrium value behave. In this way we can classify the equilibrium value as *asymptotically stable, unstable* or *semi-stable.*

The slope function can also be used to determine the concavity of solutions $y(t)$ for different values of y. In particular, a solution $y(t)$ will pass through an inflection point at some t_0 if the sign of $y''(t)$ changes at t_0. Using the Chain Rule, we can see that

$$y''(t) = F'(y(t))y'(t) = F'(y(t))F(y(t)),$$

so the second derivative $y''(t)$ also depends explicitly only on the value of y. Thus, we can speak of an *inflection value* $\hat{y}$ for $y(t)$: the solution $y(t)$ will pass through an inflection point when $y(t) = \hat{y}$ if $F'(y)F(y)$ changes sign at $y = \hat{y}$.

Take-Home Quiz Due at the Start of Class