Newton's Law of Cooling

In class we discussed a model for the cooling of an object. Our reasoning (and speculation) led to a model involving a rate equation. This is Newton's Law of Cooling:

$$H'(t) = -k\,(H(t) - A)$$

with $H(t)$, the object's *temperature* as a function of *time*, t;

$H'(t)$, the *rate* at which the temperature is changing at time t;

k, a positive constant (whose value depends on the material the object is made of, the object's shape, and the material surrounding the object, among other things).

A, the *ambient* temperature, is the temperature of the surrounding material, which is assumed to remain *constant*.

The Experiment

Just boiled water is poured into an aluminum cup sitting in an insulated ice bath. The cup is immediately covered by an insulated cap through which a thermistor (a device for measuring temperature) protrudes into the hot water. The whole ice bath is then covered, with the thermistor wire extending outside to a device which records the data. The temperature is sampled every 5 seconds. The first temperature recorded (at t = 0 seconds) is $H(0) = 84.98°C$. Empirically, the net ambient temperature (taking into account both the ice bath and the air above the ice bath) is about $A = 3.5°C$. We assume that the ambient temperature remains constant over the course of the experiment.

1. What do you think the graph of $H(t)$ will look like for $t \geq 0$? Sketch your graph below. Be sure to mark the initial and ambient temperatures on the vertical axis. Explain your reasoning, using the rate equation model and your physical experience with cooling objects as a guide.

(As this lab is progressing, we will be conducting an actual experiment under similar conditions.)

With your lab group, go to the computer and open the Excel spreadsheet `h:/Math Courses/Math110/Wk03F00Lab.xls`. The first sheet, labeled "Data," lists the recorded temperatures H and the times t at which they were recorded. Use the *Chart Wizzard* in Excel to plot the graph of $H(t)$. (Use the same procedure for constructing plots you learned in last week's lab. Save this as a separate sheet.)

2. Compare this plot with your prediction in 1. above. Is this what you expected?

Estimating the Parameter k in the Model

According to Newton's Law of Cooling,

$$H'(0) = -k \cdot (H(0) - A).$$

You know the values of $H(0)$ and A. In fact,

$$H(0) = \qquad\qquad\qquad A =$$

3. IF you had a value for the initial rate $H'(0)$, how could you determine k? Show your work, giving a formula for k in terms of $H(0)$, $H'(0)$ and A.

Recall that when the graph of a function is a straight line, the rate of change of that function is just the slope of that line. Now, even though the graph of $H(t)$ is NOT a straight line, it closely approximates a straight line near $t = 0$. To see this, use the *Chart Wizard* in Excel to look at the graph of the data on the time interval $0 \leq t \leq 30$ seconds.

4. Discuss how you can use this data to estimate $H'(0)$. State your reasoning below and give your estimate:

5. Use your estimate for $H'(0)$, your formula for k, and other information you already have to give a numerical estimate for k. (*Remark:* You may assume that temperature data is accurate to three significant figures, with the fourth figure somewhat uncertain. Accordingly, you should, in principle, keep only four significant figures throughout your calculations in estimating k.) What UNITS does k have?

Approximating the Solution of the Model

6. Begin by writing out the full initial value problem modeling this cooling cup of water, filling all parameters for which you are given or have estimated values.

7. You should now be able to fill out the first line of the table below. Discuss with your group how you are going to fill out the second and third lines, then do so. (Use $\Delta t = 10$ in each case.)

t	$H(t)$	Δt	$H'(t)$	$\Delta H \approx H'(t) * \Delta t$
0	84.98			

Have your table checked before proceeding.

8. The process you have been using is very similar to that you used in last week's lab to produce a piecewise linear approximation "Using Slopes." Recall that in that portion of the lab you were given an initial point and a formula for the slopes on each subsequent piece. In the work you have just been doing, what is the initial point? What formula are you using to get the slopes?

9. Go now to the second Excel sheet in this labs workbook, the one labelled "Model." Enter appropriate values and formulas in appropriate cells to create a piecewise linear approximation to the solution of the model by the "Using Slopes" method. Use the stepsize $\Delta t = 10$ throughout, and extend the table in the spreadsheet to a final time of 2690 seconds.

10. Use the Chart Wizard to plot this approximation together with the acual data from the first sheet. Save this plot as a new sheet. Compare these – how good is this approximation?

11. Improve your approximation by repeating this process, using a stepsize of $\Delta t = 5$ instead. Use the Chart Wizard to compare this approximation with your previous approximation. Is it significantly better?

12. If necessary, choose even smaller sizes for Δt until you are satisfied that you cannot significantly improve your approximation. Use the Chart Wizard to save your best result. Also note which stepsize you use.

    SAVE YOUR WORK ON DISKETTE!

Preparing Your Lab Report

Your report should consist of a cover page with the title of the report and the names and signatures of your lab group members. **Also indicate your Lab Section.** Each person (in a group of three) should complete a first draft of one of the three parts, and the group should meet to read and discuss these drafts before submitting the final report. The final report is DUE IN NEXT WEEK'S LAB. Grading will be Credit/No Credit with, however, a high standard for receiving credit. If you do not initially receive credit you will have one week to revise the report to correct any problems with it.

You will need to refer to your Excel worksheets for this lab to answer these questions.

Part 1

State the initial value problem you worked on in this lab. Include a complete explanation of how you estimated the parameter k. Also discuss assumptions behind this model, how realistic they are, and any other limitations of the model that you think may be important.

Part 2

Carefully explain the procedure you used to obtain piecewise linear approximations to the solution of the initial value problem. What happened as you decreased the stepsize? How well did the model account for the actual experimental data? Illustrate your discussion with graphs, if possible.

Part 3

You may suspect that the model solution or the experimental data (or possibly both) can be fit by an exponential function. Use techniques from class and homework to see whether this is true, and show all your work.