Objectives:

1. To become more familiar with the program **Derive** as a tool..
2. To determine differentiability of a function at a point by taking limits of difference quotients.
3. To better understand the relationship between differentiability and continuity.

§1 Introduction: Using *Derive* To Explore Graphs

Derive has rather nice features for exploring graphs of functions. This is what we will use it for today. To begin, click on the *Derive* icon under *Mathematics*. A screen will appear with a list of menu options and buttons at the top. This particular screen is called the "Algebra" window in *Derive* because this window will be used to author and modify algebraic and other expressions.

Type

$$\sin(1/x)$$

in the authoring window, then $<$ *Enter* $>$ it. The authoring window will disappear and your expression will appear in the algebra window.

To plot the function whose rule is given by an expression, make sure the expression is *highlighted* in the algebra window, then click on the second button from the right in the toolbar. Do this now.

The screen is now replaced with a "graphics" window. This window will have a pair of axes marked with tickmarks and its own menu at the top. Now select and enter **Plot** from this menu. The graph of this function should appear.

Derive has several features which allow you to explore graphs. First, notice the cross-hairs. They can be controlled with either the mouse or the "arrow" keys. At the bottom of the screen you will see the x- and y-coordinates changing as you move the cross-hairs around.

Now examine the bottom of the screen more closely. The spacing between the tickmarks on the x and y axes will appear as **Scale** in the format *x-scale*: *y-scale*. What are these values now?

There are several features of the graphics window menu which we will also be using. Select **Set**, then **Center**. Type 0 for the *Horizontal* coordinate and 1 for the *Vertical* coordinate, then $<$ *Enter* $>$. Describe what happens. (Also note the *Center* box at the bottom of the screen.)

The other feature we will be using is **Zoom**. Various sorts of zooming are possible. These are performed by the buttons at the right side of the menu bar with little arrows on them. Find and select the button which *zooms in on both axes*. Describe what happens. Pay particular attention to the values for the x and y scales.

You now know the basics of working with *Derive*. During the rest of the lab, we will be using the following sequence of operations to focus on certain points of the graph of a function.

Move the cross-hairs to the point of interest.

Center *the window on that point.*

Zoom *in on the center of the window.*

Try zooming in and out on various points just to get the hang of this sequence of operations.
Recall
Looking at your "zoomed-in" plot of $\sin(x)/x$ does $\lim_{x\to 0^+} \sin(1/x)$ exist?

You will probably have to recall the definition of a limit to answer this question.

§3 Differing Difference Quotients

We have previously defined the derivative of $f(x)$ at a point a using the *forward difference formula*

$$f'(a) = \lim_{h\to 0^+} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0^-} \frac{f(a+h) - f(a)}{h}$$

If these two limits exist and are equal then we say that $f'(a)$ exists and is equal to these limits. IF you know that $f'(a)$ exists, you can also calculuate it using the more accurate *centered-difference formula*

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a-h)}{2h}.$$

Use Derive to graph $y = f(x) = x^3 - 1$ on with an x range of $[0, 2]$ and a y-range of $-2, 2]$.

We will use the table below to investigate what happens when you find the derivative of $f(x) = x^3 - 1$ at $x = 1$ using the different difference quotients. You can use Derive to help you complete the table by defining a function. If you *Author* `RIGHT(h) := (f(1+h) - f(1))/h` (don't forget the colon) then you now have a function `RIGHT(h)` which you can use to complete the first table below. Similarly define a `LEFT(h)` function to help you complete the second table.

h	$1+h$	$f(1+h)$	$\frac{f(1+h)-f(1)}{h}$
0.1			
0.01			
0.001			
0.0001			

h	$1+h$	$f(1-h)$	$\frac{f(1+h)-f(1)}{h}$
-0.1			
-0.01			
-0.001			
-0.0001			

Use your answers from above to help you complete the following table.

h	$\frac{f(1+h)-f(1-h)}{2h}$
0.1	
0.01	
0.001	
0.0001	

Do you expect the results of the three limits to agree? Do they? What is the value of $f'(1)$ for $f(x) = x^3 - 1$?

Write down the equation of the tangent line to $f(x) = x^3 - 1$ at $x = 1$.

Use *D*erive to show the function $f(x)$ and its tangent line on the same graph. Zoom in on the point where the tangent line and curve intersect. Can you zoom in enough so that the difference between the two graphs is negligible? What does this tell you about the relationship between *local linearity* and differentiability?

§3 The Derivative of the Absolute Value Function

The absolute value function is denoted by "abs(x)" in *Derive*. Use *Derive* and the methods you have just learned to first obtain a graph of $y = f(x) = |x|$, and then estimate the slope of the graph at $x = 0$.

h	$\frac{f(0+h)-f(0)}{h}$	$\frac{f(0-h)-f(0)}{-h}$	$\frac{f(0+h)-f(0-h)}{2h}$
0.1			
0.01			
0.001			

Do you think it is possible to define the slope of the graph of the absolute value function at $x = 0$? Why or why not? How does this result affect your understanding of the relationship between differentiability and continuity?

§4 Using Derive to Take Limits

Consider the difference quotients when $f(x) = |x|$. Simplify the expressions algebraically an dthen write down the *Derive* formula for the simplified form of the difference quotients.

$$\frac{f(0+h) - f(0)}{h} =$$

$$\frac{f(0+h) - f(0-h)}{2h} =$$

If you *Author* each expression in *D*erive and then select the `Lim` button you will get a window which allows you to take limits with respect to different variables and from either the "left," "right" or "both." Be careful you understand what variable is used in each of the examples above. Take the limit from the left and then the right and check to see how they correspond with the limit from "both" for each difference quotient above. Does this confirm the results you obtained using the "tabular method" of computing limits?

Preparing Your Lab Report

Your report should consist of a cover page with the title of the report and the names and signatures of your lab group members. **Also indicate your Lab Section, Lab Time and Instructor**, e.g. Section 2: Thursday 8:30 am (Buckmire). Each person (in a group of three) should complete a first draft of one of the three parts, and the group should meet to read and discuss these drafts before submitting the final report. The final report is DUE IN NEXT WEEK'S LAB. Grading will be Credit/No Credit with, however, a high standard for receiving credit. If you do not initially receive credit you will have one week to revise the report to correct any problems with it. If you have questions whether your draft will likely receive Credit, you should visit your instructor in office hours and have your draft lab report evaluated.

Your lab report should take care to answer the following *specific* questions. Be sure to provide the context for each question in your answer.

Part 1
Consider the graph of $\sin(1/x)$. Is $\sin(1/x$ continuous at $x = 0$? Is it differentiable at $x = 0$? Give evidence to support your answers.

Part 2
Consider the expressions for `RIGHT(h)`$= \dfrac{f(1+h) - f(1)}{h}$ and `LEFT(h)`$= \dfrac{f(1+(-h)) - f(1)}{-h}$. Show that `LEFT(h) = RIGHT(-h)`. Also, show that the average of `LEFT(h)` and `RIGHT(h)` is exactly the centered-difference formula, `CENTER(h)` $= \dfrac{f(1+h) - f(1-h)}{2h}$.
Why is it that the derivative at a point is not defined using the centered-difference formula? (HINT: What evidence do you have from this lab that using the centered-difference formula to find the derivative does not always give you the correct answer? Think about the behavior of the absolute value function at $x = 0$.)

Part 3
Consider the rate of convergence of the centered-difference formula to the derivative. Does your tabular evidence support the claim that taking the limit using this formula converges to the derivative faster than the difference quotient used to define the derivative does? Do you believe this statement is true? Try graphically depicting what each method looks like for the function $f(x) = x^3 - 1$ at $x = 1$ to support your claim.