
Mathematics As A Liberal Art

Math 105 Spring 2024

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Fowler 309 MWF 3:00pm- 3:55pm

<http://sites.oxy.edu/ron/math/105/24/>

Class 14: Friday March 1

Mathematical Logic: Conclusions

DEFINITION: Syllogism

A **syllogism** is a type of logical argument consisting of two or more propositions (premises) and a deduction (inference). The conclusion of the syllogism is inferred to be true by assuming the premises are true. This form of logical argument is most closely associated with the Greek philosopher Aristotle and typically looks like:

major premise

minor premise

∴ conclusion

DEFINITION: Hypothetical Syllogism

The hypothetical syllogism is one of the most common examples of valid arguments made in mathematics. It is an argument that has the mathematical form

$$(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow p \rightarrow r$$

In English sentences, this argument has the form:

IF the sun is shining THEN you will get a sunburn.

If you get a sunburn, then you will be in pain.

THEREFORE: If the sun is shining, then you will be in pain.

Exercise

Write down your own example of a hypothetical syllogism below:

DEFINITION: Disjunctive Syllogism

The disjunctive syllogism is another one of the most common examples of valid arguments made in mathematics. It is an argument that has the mathematical form

$$(p \vee q) \wedge \neg p \rightarrow q$$

EXAMPLE

We can show by using a truth table that one obtains a tautology when one evaluates the disjunctive syllogism, i.e. the output of this statement is TRUE for all values of the Boolean variables p and q .

p	q	
T	T	
T	F	
F	T	
F	F	

Exercise

Write down your own example of a disjunctive syllogism below:

Examples of Invalid Arguments (Fallacies)**Hypothetical Denial**

This fallacy is also known as the “fallacy of the inverse” or “denying the antecedent” or simply **inverse error**. Mathematically, it can be represented as

$$(p \rightarrow q) \wedge \neg p \rightarrow \neg q$$

Consequential Affirmation

This fallacy is also known as the “fallacy of the converse” or “affirming the consequent” or simply **converse error**.

$$(p \rightarrow q) \wedge q \rightarrow p$$

DEFINITION: Fallacy

A **fallacy** is a logical argument that is not valid, because it can be shown to not always be true (i.e. it is not a tautology).

GROUPWORK

Write down your own example of “denying the antecedent” below:

Write down your own example of “affirming the consequent” below:
