Class 7: **Wednesday February 7**

Introduction to Algorithm: Many Multiplications

SUMMARY

In today's class we will look at multiplication and come up with two different **algorithms** that can be used to compute the product of 2-digit numbers.

DEFINITION: **algorithm**

An **algorithm** is a step-by-step set of instructions or process to be followed in order to complete a calculation or solve a problem; it is usually done by a computer.

The Method of Doubling

Suppose you want to multiply 26×78

One way to do this is to use the **Method of Doubling**. Choose the larger number of the two products, and double it, and keep doubling it until if you doubled it again you would exceed the product you are trying to compute.

$$
\begin{aligned}
78 \times 1 &= 78 \\
78 \times 2 &= 156 \\
78 \times 4 &= 312 \\
78 \times 8 &= 624 \\
78 \times 16 &= 1248
\end{aligned}
$$

You stop doubling now because the next double would be 78×32 which is larger than the number you are looking for, 78×26.

Now, look closely at 26. Since $26 = 16 + 8 + 2 = 2^4 + 2^3 + 2^1$. Thus,

$$
\begin{aligned}
78 \times 26 &= 78 \times (16 + 8 + 2) \\
&= 78 \times 16 + 78 \times 8 + 78 \times 2 \\
&= 1248 + 624 + 156 \\
&= 2028
\end{aligned}
$$

The nice thing about this algorithm is that it means that you only have to know how to multiple something by 2, you don't need to remember all those other messy entries in the multiplication table! **The method of doubling works because any whole number can always be written as a the sum of powers of 2.**

EXAMPLE

Let's evaluate 26×78 by the method of doubling again (but this time we will express 78 as a sum of powers of 2).

Exercise

Evaluate 39×51 by the method of doubling.

Another Means of Multiplication: the Grid Method

In the grid method you separate the numbers to multiplied into their component parts: hundreds, tens and units. So, 26 becomes 20 and 6 and 78 becomes 70 and 8. You multiply 20 times 70 and 20 times 8 and then 6 times 70 and 6 times 8 and place them in the correct spots. Then you add up everything in the columns and then to get the final answer you add up the numbers in the bottom row: **1560 + 460 = 2028**.

	20	**6**	**×**
	1400	420	**70**
	160	48	**8**
	1560	468	**2028**

Exercise

Evaluate 39×51 by the grid method.

Another Means of Multiplication: the Lattice Method
The "Usual" Way To Multiply Two 2-digit Numbers

Usually if we were going to multiply 26×78 we would multiply **8 times 6** get 48, right down the 8, then put the 4 above the 2 in the "tens" column, then multiply **8 times 2** and get 16, add the 16 plus the 4 to get 20 and write down 208 in the first row.

Then you repeat the algorithm by multiplying **7 times 6** to get 42 right down the 2 in the tens column, and write the 4 above the 2 in 26 (tens column). Continue by multiplying **7 times 2** to get 14 and add the 14 plus the 4 from above the 2 to get 18 and write down 18 in the hundreds column.

```
            2   6
    ×
            7   8
    ─────────────
        2   0   8
    1   8   2
    ─────────────
    2   0   2   8
```

Notice that there were 4 actual multiplication operations you had to do two multiply the two 2-digit numbers 78×26: **8 times 6**, **8 times 2**, then **7 times 6** and **7 times 2**. The rest of the multiplication algorithm is bookkeeping (making sure you add the proper ones, tens, and hundreds together correctly. To do this we draw in diagonal lines that look like "/" in every square of the lattice. Then when you write down the two digit product, like 42, you write the 4 on the left of the slash and the 2 on the right, like "4 / 2." Then when the lattice is completely filled in you add your single digit answers along the diagonals, so you should get (in order from right to left): 8, then 6+4+2=12 (so you write down a 2 and carry the ten as a 1 to the next diagonal) and then add 1+1+4+1=10 (so you write down 0 and carry the ten as a 1 to the last diagonal) and add 1+1 to get 2. So, from right to left you should get 8 then 2 then 0 then 2, or 2028.

	2	**6**	×
			7
			8

The lattice method works is accomplished by keeping very close track of which of the products involve "ones by ones" and "ten by ones" and "ones by tens" and then "tens by tens" (i.e. hundreds).

$\boxed{\text{EXAMPLE}}$
Evaluate 39×51 by the lattice method.

GROUPWORK

Compute 27×68 using (a) the method of doubling (b) the grid method (c) the lattice method.

CHALLENGE

Try using the grid method OR the doubling method OR the lattice method to compute 123×47.

EXTRA CREDIT (5 points)

Does everyone need to know how to multiply 2-digit numbers in the age of calculators built into the hand-held devices we take everywhere? If not multiplication, then how about addition? Subtraction? Fractions? Percentages? What is the minimum amount of "arithmetic" (number crunching) does the average person need to know? Should there be a standard, and if so, what should it be? **Write at least 250 words (max: 500) responding to these questions and others in order to express your response to the question of whether everyone needs some level of facility with numerical calculations (and how much).**