Mathematics As A Liberal Art

Math 105 Spring 2024 Fowler 309 MWF 3:00pm- 3:55pm

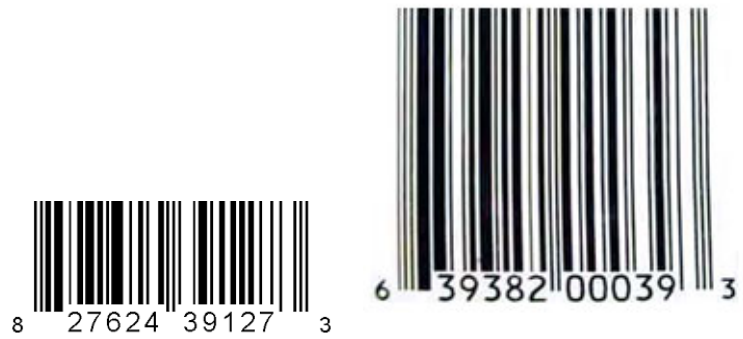
 http://sites.oxy.edu/ron/math/105/24/

Class 6: Friday February 2

Looking For Numbers In Patterns

SUMMARY

Previously we have looked for patterns in numbers and discovered it is pretty common to find patterns in sequences of numbers. Today we are going to look at the world around us and see how omnipresent numbers are, especially in the commercial products we buy.



Applications of Modular Arithmetic: Universal Product Codes

A **Universal Product Code** is a numerical code which is used to identify and track products in countries such as the United States, Canada, United Kingdom and Australia. It uses a barcode that can be rad by a machine along with a digital code that has a check digit.

Given a 12-digit code of the form $d_1d_2d_3d_4d_5d_6d_7d_8d_9d_{10}d_{11}c$ we can determine whether this number is a valid Universal Product Code if the digits follow the formula:

$$3d_1 + d_2 + 3d_3 + d_4 + 3d_5 + d_6 + 3d_7 + d_8 + 3d_9 + d_{10} + 3d_{11} + c \equiv 0 \pmod{10}$$

EXAMPLE

Show that the given UPC if 639383 000393 is indeed a valid product code.

Exercise

Determine whether the other UPC (827624 391273) given above is also a valid UPC.

The Beauty of The Check Digit!
Consider the following UPC: 077043 103296.
We can show that this is a legitimate UPC by applying the formula.

GroupWork
Transpose (swap) any of the digits (except the last one), for example: 707043 103296 or 077034 103296 or 077034 130296 and show that the transposed codes are INVALID codes because the check digit will not work.

Transpose another two numbers and check it again!

Numbers Are Everywhere!

There are many other encoding schemes with check digits which use different weighting vectors corresponding with other coding systems, like the Vehicular Identification Number (17 alphanumeric code) **VIN** , Bank Identification Numbers (9 digits), (Credit Cards (16 digit codes) **Mastercard/Visa** and the International Standard Book Number **ISBN** (10 digits or 13 digits).

International Standard Book Number

Before January 1, 2007 most books had 10-digit codes called ISBN-10 and used the following system for evaluating their validity

$$x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 + 6x_6 + 7x_7 + 8x_8 + 9x_9 + 10x_{10} \equiv 0 \pmod{11}$$

The new code is called an ISBN-13 and the first three digits are 978 but the validation formula has changed to

$$x_1 + 3x_2 + x_3 + 3x_4 + x_5 + 3x_6 + x_7 + 3x_8 + x_9 + 3x_{10} + x_{11} + 3x_{12} + x_{13} \equiv 0 \pmod{10}$$

Many books have both an ISBN-10 and an ISBN-13 printed on them.

Keith Devlin's *Mathematics: The Science of Patterns* has the ISBN-13 code: **9-780716-760221** and the ISBN-10 code **0-7167-6022-3**.

Exercise

Verify that **9-780716-760221** is a valid ISBN-13 code.

Verify that **0-7167-6022-3** is a valid ISBN-13 code.

How UPC bar codes work

Each number consists of four bars of alternating light and dark which are either single (1), double (2), triple (3) or quadruple (4) in width. The barcode begins and ends with three bars of shortest width, i.e. (1-1-1).

0	1	2	3	4	5	6	7	8	9
3-2-1-1	2-2-2-1	2-1-2-2	1-4-1-1	1-1-3-2	1-2-3-1	1-1-1-4	1-3-1-2	1-2-1-3	3-1-1-2

(Source: How Stuff Works website)

We should be able to take the UPC 6-39382-00039-3 and write down the alternating widths of light and dark bars we will see the following. (NOTE: we have ignored the 1-1-1 bars at the beginning and end as well as the 1-1-1-1-1 in the middle separating the two halves of the code.)

6	3	9	3	8	2	0	0	0	3	9	3
1-1-1-4	1-4-1-1	3-1-1-2	1-1-1-4	1-2-1-3	2-1-2-2	3-2-1-1	3-2-1-1	3-2-1-1	1-4-1-1	3-1-1-2	1-4-1-1

GROUPWORK

Can you decipher what UPC code this is? [HINT: Try and figure out the widths of the dark bars and write them on the bottom and write the widths of the lights bars and write them on the top.)

POTENTIAL PROJECT TOPIC

A potential topic for a project could be explaining how QR codes (which are essentially 2-dimensional bar codes) work or looking at how Vehicle Identification Numbers or some other ubiquitous numerical code system works!