Class 5: Wednesday January 31

Mathematical Induction

Proving A Pattern Is True: Producing A Formula

Consider the following sequences of numbers in the table below

The Integers	1	2	3	4	5	6	...	n
Sum of (First Number Of) Odds	1	4	9	16			...	n^2
Sum of (First Number Of) Evens	2	6	12	26			...	n^2+n
Sum of (First Number Of) Integers	1	3	6	10			...	$\frac{n(n+1)}{2}$
Sum of (First Number Of) Squared Integers	1	4	9	16			...	$\frac{n(n+1)(2n+1)}{6}$
Sum of (First Number Of) Cubed Integers	1	9	36	100			...	$\frac{n(n+1)^2}{2}$

Mathematical Induction

Mathematicians use a process called **mathematical induction** to prove that a formula $P(n)$ that looks like it works for all n values actually does so.

Mathematical induction involves two steps.

STEP 1: **Show that the Basis Step is True**. In other words, show that $P(1)$ is true. In other words, demonstrate that the pattern you are looking at represented by $P(n)$ does indeed work for some small case, like $n = 0$ or $n = 1$.

STEP 2: **Show that the Inductive Implication is True**. In other words, prove that $P(n + 1)$ is true by assuming that $P(n)$ is true. In mathematical symbols, we say that $P(n) \Rightarrow P(n + 1)$.

The reason that Mathematical Induction works is if you consider what you have done like flipping an infinite row of dominoes. You have shown that the statement is true for the basis step. And then you have shown that if the statement is true for one value, it will be true for the next value. Which means that it will be true for the next value after that, and the one after that, for ever, in other words for EVERY value.

Mathematical induction is just one example where mathematics can be used to formally describe and explain a pattern that occurs in nature.

EXAMPLE

Use Mathematical Induction To Prove That The Sum Of The First N Even Integers Is $N^2 + N$.

GROUPWORK

Use Mathematical Induction To Prove That The Sum Of The First N Odd Integers Is N^2.

BASIS STEP

INDUCTIVE STEP

More practice with Mathematical Induction

1. Use mathematical induction to show that $n < 2^n$ for all numbers $n \geq 1$

BASIS STEP

INDUCTIVE STEP

2. Use mathematical induction to show that

$$1 + 2 + 2^2 + 2^3 + 2^4 + \ldots + 2^n = 2^{n+1} - 1$$

for all integers $n \geq 0$.

BASIS STEP

INDUCTIVE STEP

3. Use Mathematical Induction to verify one of the formulas in the table (from page 1) that you have not verified yet. (They are all true for $n \geq 1$)