Mathematics As A Liberal Art

Math 105 Spring 2024 Fowler 309 MWF 3:00pm- 3:55pm

 http://sites.oxy.edu/ron/math/105/24/

Class 4: Monday January 29

Looking For Patterns In Numbers

The Seven Original Liberal Arts

There were SEVEN fundamental liberal arts of the followers of Pythagoras: **arithmetic** (the study of numbers), **geometry**, **music** and **spherics** (what we call astronomy or the study of the spheres in the sky), **grammar**, **logic** and **rhetoric**. In the Middle Ages, the first group of four were grouped together and called the **quadrivium** while the rest where known as the **trivium**. These seven liberal arts came to be looked upon as the necessary equipment of an educated person. (Adapted from page 76 of the 6th edition of *An Introduction to the History of Mathematics with Cultural Connections* by Jamie H. Eves, Brooks/Cole: Pacific Grove, CA, 1990.)

Friendly Numbers, Perfect Numbers, Deficient Numbers, Abundant Numbers

The Pythagoreans imbued their study of numbers with all sorts of mysticism, and distinguished between **arithmetic**, the abstract relationship between numbers, and **logistic**, the pacifical art of computing with numbers.

DEFINITION: amicable or friendly numbers

Two number are said to be **amicable** or **friendly** if each is the sum of the proper divisors of the other.

EXAMPLE: 284 and 220 are friendly numbers.

DEFINITION: proper divisor

The proper divisors of a positive integer N are all the positive whole numbers that divide N exactly except for N itself. NOTE: 1 is always a proper divisor of N! (So is N). Another (archaic) term for proper divisor is *aliquot part*.

DEFINITION: perfect, deficient and abundant numbers

A number is said to be **perfect** if it is the sum of its proper divisors. The number is **deficient** if it is greater than the sum of its proper divisors and **abundant** if it is less than the sum of its proper divisors.

EXAMPLE: 6 is a perfect number, 8 is a deficient number, 12 is an abundant number.

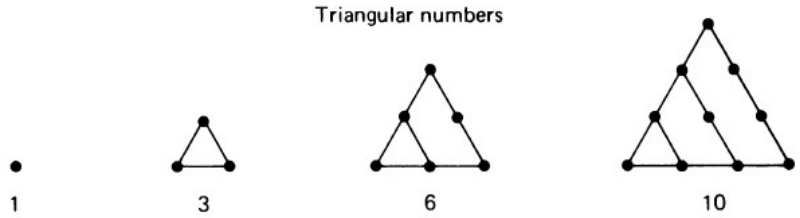
DEFINITION: prime number

A number is said to be **prime** if it is a positive integer greater than 1 without having any positive whole number divisors except for itself. EXAMPLE: The first few prime numbers are: 2, 3, 5, 7, 11, ...

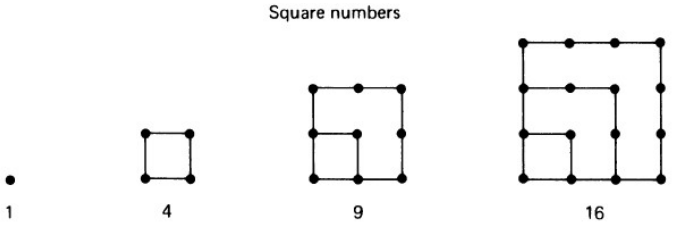
Figurate Numbers

The Pythagoreans originated the concept of the "figurate" numbers. These are sequences of numbers associated with placing a dot in a geometric pattern and then increasing the number of dots to maintain the pattern. This represents a link between geometry and arithmetic that the Pythagoreans would have greatly appreciated.

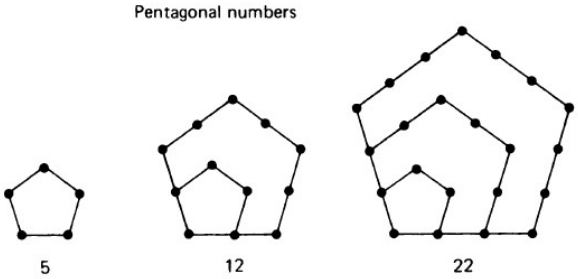
Triangular Numbers T_n



Square Numbers S_n



Pentagonal Numbers P_n



GROUPWORK

Extend the list of triangular, square and pentagonal numbers. Do you notice any patterns? What comes next? Can you find a formula for the n^{th} number in each of these sequences?