Theorems about Prime Numbers

1. There are an infinite number of primes.
2. If a prime p divides a product mn then p divides at least one of m or n.
3. (**Fundamental Theorem of Arithmetic**)

   Every natural number is prime or can be expressed as a product of primes that is unique apart from the order in which the product is written.
4. (Hadamand/Poussin) **The Prime Number Theorem**

   Considering that $\pi(N)$ is the **prime density function**, the proportion of the first N natural numbers which are prime numbers behaves like

   $$\frac{\pi(N)}{N} \to \frac{1}{\log N} \text{ as } N \to \infty$$
5. (Chebychef) There is always a prime number between N and $2N$.
6. (**Fermat's Little Theorem**) If a is a whole number and p is a prime that does not divide a, then $a^{p-1} \equiv 1 (\text{mod } p)$
7. A number of the form $M_n = 2^n - 1$ is called a Mersenne prime. We know that M_n prime $\Longrightarrow n$ is prime. The converse is not true. (In other words, it is not true that if n is prime then $M_n = 2^n - 1$ is prime.)

Conjectures about Prime Numbers

1. **Goldbach Conjecture**: Every even integer greater than 2 is the sum of two primes.
2. **Twin Primes Conjecture**: There are an infinite number of twin primes.
3. **Mersenne Primes Conjecture**: There are an infinite number of Mersenne primes.

Fermat's Last Theorem (Proved by Andrew Wiles in 1993-94)

$$x^n + y^n = z^n$$

has no solutions for $n > 2$!