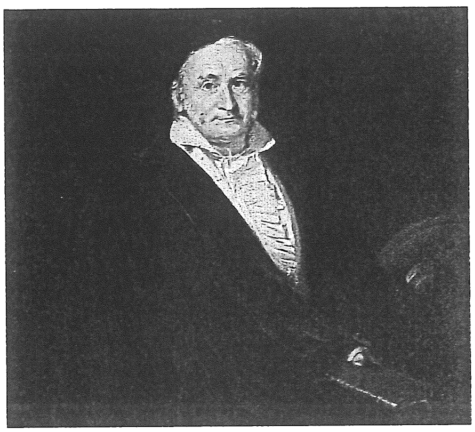
Gauss: The Child Genius



Karl Friedrich Gauss (1777–1855).

Born in Brunswick, Germany, in 1777, Karl Friedrich Gauss displayed immense mathematical talent from a very early age. Stories tell of him being able to maintain his father's business accounts at age three. According to another story, while in the elementary school, Gauss confounded his teacher by observing a pattern that enabled him to avoid a decidedly tedious calculation.

Gauss' teacher had asked the class to add together all the numbers from 1 to 100. Presumably the teacher's aim was to keep the students occupied for a time while he was engaged in something else. Unfortunately for him, Gauss quickly spotted the following shortcut to the solution.

You write down the sum twice, once in ascending order, then in descending order, like this:

$$1 + 2 + 3 + \cdots + 98 + 99 + 100$$

$$100 + 99 + 98 + \cdots + 3 + 2 + 1.$$

Now you add the two sums, column by column, to give

$$101 + 101 + 101 + \cdots + 101 + 101 + 101.$$

There are exactly 100 copies of the number 101 in this sum, so its value is

$$100 \times 101 = 10{,}100.$$

Since this product represents twice the answer to the original sum, if you halve it you obtain the answer Gauss' teacher was looking for, namely 5050.

Gauss' trick works for any number n, not just 100. In the general case, when you write the sum from 1 to n in both ascending and descending order and add the two sums column by column, you end up with n copies of the number $n + 1$, which is a total of $n(n+1)$. Halving this total gives the answer:

$$1 + 2 + 3 + \cdots + n = n(n + 1)/2.$$

This formula gives the general pattern of which Gauss' observation was a special case.

It is interesting to note that the formula on the right-hand side of the above identity also captures a geometric pattern. Numbers of the form $n(n + 1)/2$ are called *triangular* numbers, since they are exactly the numbers you can obtain by arranging balls in an equilateral triangle. The first five triangular numbers, 1, 3, 6, 10, 15, are shown below.