

Name \_\_\_\_\_

**Geometric Series**

Look at this infinite series:

$$S = 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 \dots$$

1. We ask: Does this series have a sum? Multiply  $S$  through by  $-1$  to get  $-S$ . Write down the expression for  $-S$ . What happens?

Now subtract  $-S$  from  $S$  to get  $2S$ . What happens to the terms of the series? Show how this allows you to “prove” that the sum is  $1/2$ .

2. Now have a look at  $S$  like this:

$$S = (1 - 1) + (1 - 1) + (1 - 1) + (1 - \dots$$

What do you think the sum is now?

Can you think of a way to group the elements so that the sum may be thought to be 1? Explain.

If you can arrange the terms in different ways and get different answers for the sum of the series what do you think the “actual” sum of the series is?

3. Here's another series. Our goal is to determine the sum of this series.

$$T = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$

What is the pattern here?

Is  $T$  an example of a geometric series? Then what is  $r$ , the ratio of consecutive terms? What is THE sum  $S_\infty$ ?

4. Here's our third infinite series

$$U = 1 + \frac{5}{3} + \frac{10}{6} + \frac{15}{9} + \dots$$

What is the pattern here?

Is  $U$  an example of a geometric series? Then what is  $r$ , the ratio of consecutive terms? What is THE sum  $S_\infty$ ?

5. Discuss the convergence/divergence behavior of the series  $S$ ,  $T$  and  $U$ . How do you know which series converge or diverge? EXPLAIN YOUR ANSWERS!