Name _____

Applications of Modular Arithmetic

1. UPC (The Universal Product Code)

The UPC is usually made up of a 12 digit number. The first six digits encode information about the manufacturer, and the next five digits encode information about the product. That leaves us with the last digit, which is called the **check digit**. This check digit allows us to detect if we have an incorrect UPC number using mod 10 arithmetic. Call the digits d_1 through d_{11} and c (for "check"). Then the UPC must satisfy the following formula:

$$3d_1 + d_2 + 3d_3 + d_4 + 3d_5 + d_6 + 3d_7 + d_8 + 3d_9 + d_{10} + 3d_{11} + c \equiv 0 \ (\mathrm{mod}\ 10)$$

  1. Is 036000 285109 a legitimate UPC?

  2. What about 360000 285109? Is this a legitimate UPC?

  3. Find something else with a 12-digit UPC. Give the UPC code and verify that it satisfies the equation.

     Item: _____   UPC Code: _____

2. ISBN (International Standard Book Number)

This 10 digit code is assigned by the publisher. These ten digits (call them $x_1, ..., x_{10}$) consist of blocks identifying the language, the publisher, the number assigned to the book by its publishing company, and finally a 1 digit check that is either a digit or the letter **X** (used to represent the number 10). This check digit is selected so that

$$x_1 + 2x_2 + 3x_3 + ... + 10x_{10} \equiv 0 \ (\text{mod } 11)$$

and is used to detect errors in individual digits and transposition of digits.

1. The first nine digits of an ISBN code are **0-0705-3965**. What is the check digit for this book?

2. One of the ISBN numbers for Devlin's *Mathematics: The Science of Patterns* is **0-7167-6022-3**. Confirm that this ISBN satisfies the modular formula above.

3. Here is the ISBN number where one of the digits has been accidentally replaced by the letter Q: **0-201-57Q89-1**. Find the correct value to put in place of Q.