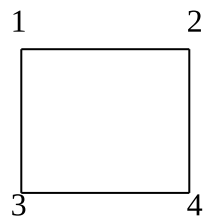
Problem Set #4
(60 points)

Name: ____________________________ Score: _______/60

Work on the following problems to turn in **Monday April 22**. Please make sure your work is clear, neat, and organized. A reminder: You may discuss these problems with each other and solve them collaboratively, but your write-up and your submission must represent **your own work written independently of others**.

1. ***(10 points)*** **Symmetries of the Square**. On this page list the eight (8) symmetries of the square. Begin with the following figure:

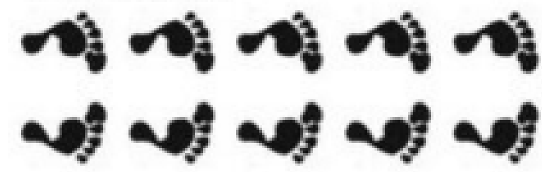
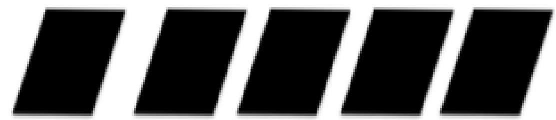
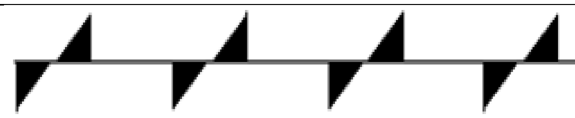
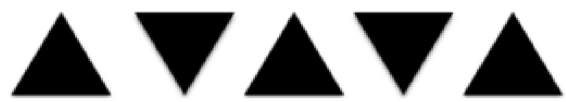
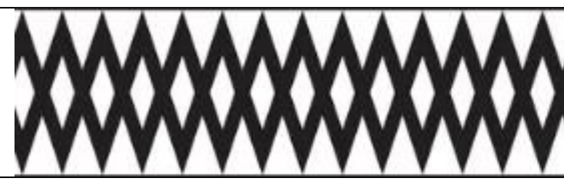


(i) Write a description (e.g., "clockwise rotation of 90 degrees about the center") of each symmetry, then (ii) draw what the square above would be transformed to under that symmetry. Clearly number/label the eight symmetries and make sure I can tell which figure goes with which written description. **You should indicate which of your transformations are involutory (they are their own inverse).**

2. ***(10 points)*** **Frieze Patterns**.

Mark with an **X** all the symmetries that each frieze exhibits. If the pattern you choose does not have the symmetry leave that space blank.

Frieze Choice	**A**	**B**	**C**	**D**	**E**
Translation					
Vertical Line Reflection					
Horizontal Line Reflection					
180º Rotation					
Glide Reflection					

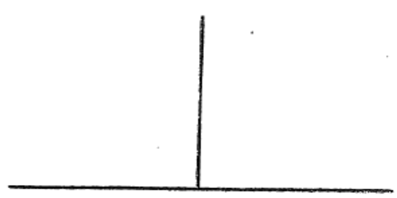
Frieze A	
Frieze B	
Frieze C	
Frieze D	
Frieze E	

3. ***(40 points)*** **An Exploration of Fractal Geometry**. While we focused on 2-, 3-, and 4-dimensional spaces and objects in class, there is also a notion of ***fractional*** dimensions! This idea comes into play in the study of what are called "fractals" The purpose of this set of problems is to let you explore the idea of fractals, although we won't get you to the idea of dimension here.

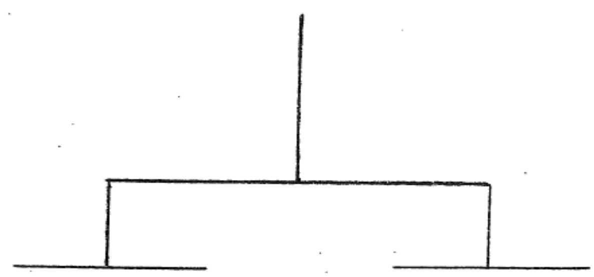
Some basic information on Fractal Geometry:

- In 1961 Benoit Mandelbrot really began the study of fractals in earnest.
- A fractal is a geometric figure that consists of an identical "motif" repeating itself over and over again, but on an ever-reduced scale.
- Fractals can be seen in nature – from the jagged edges of a coastline, to the intricate patterns of a snowflake, to the complex formation of fern leaves.

Example A. **Tree Fractal**. *(10 points)* The motif here is an inverted T. I've shown the first three stages of the construction of this fractal. Draw in the next stage.



Stage 1



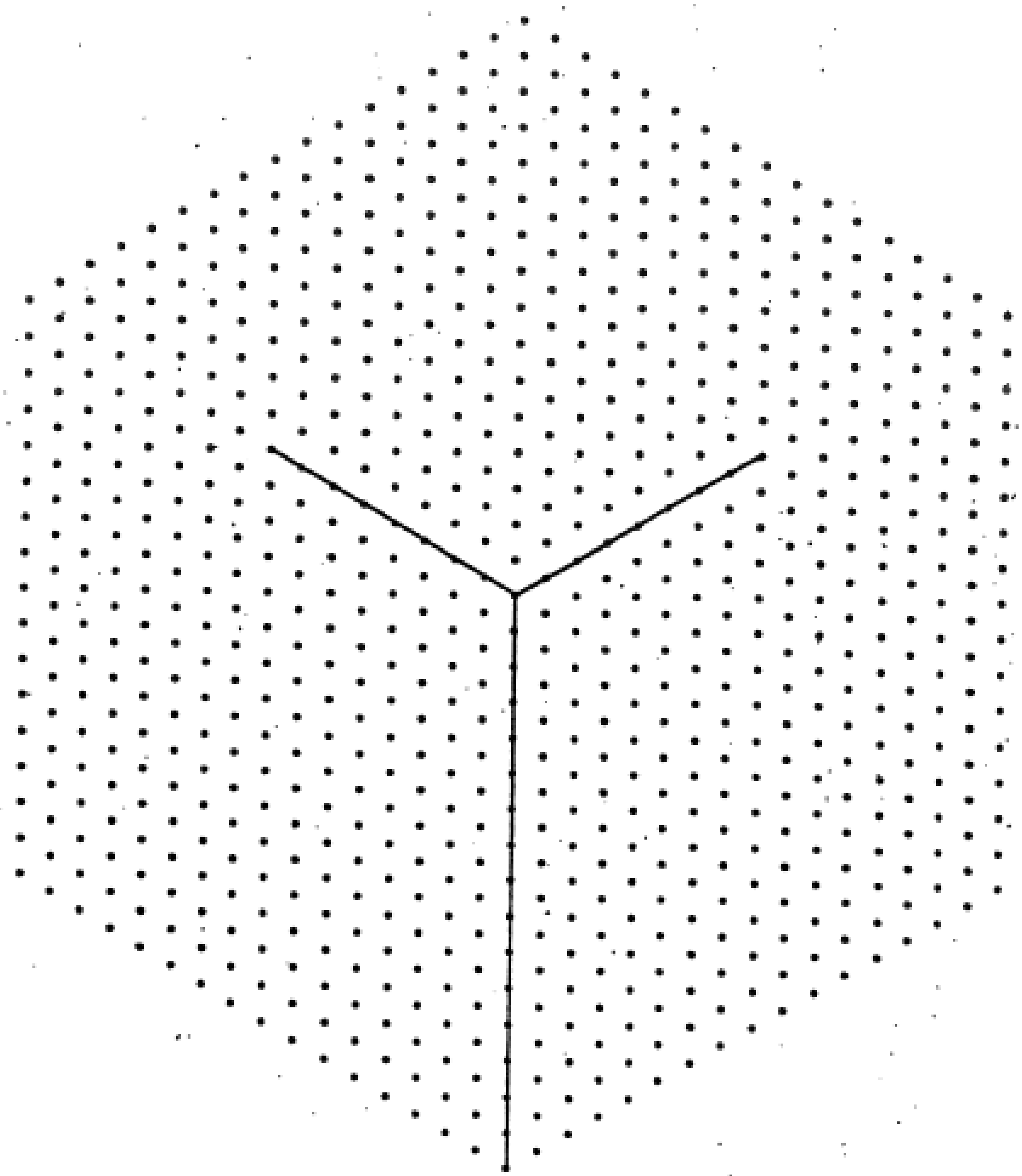
Stage 2

Stage 3

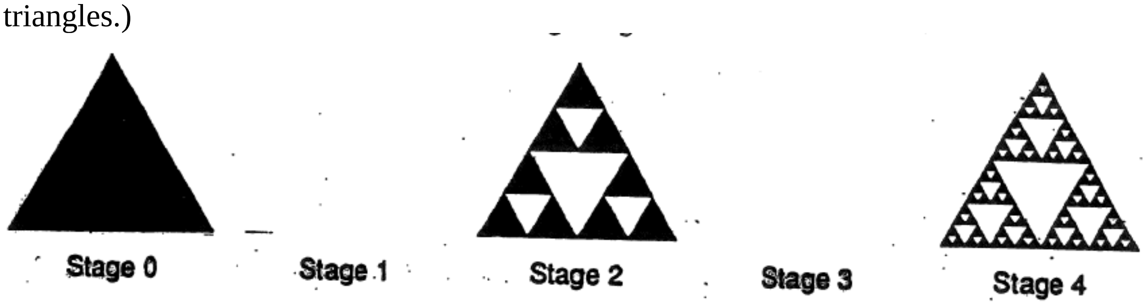
Stage 4

Those of you interested in the NCAA basketball tournament, might see this as part of the brackets for a single-elimination tournament!

Example B. *(10 points)* **Variation on the Tree Fractal**. Here we draw the next stage by growing two branches, half as long as the previous branch, off the endpoint of each branch from the previous stage. We do so at 120-degree angles from the branch. Stage 1 (1 branch of length 16) and Stage 2 (2 branches of length 8) are drawn in for you. Draw in the details for the next three stages (with branches of length 4, 2, and 1 respectively).



<u>Example C</u>. *(20 points total)* **Sierpinski Triangle**. Three of the first five stages (starting at stage 0) are drawn below. Fill in the two missing stages with careful drawings. In each subsequent stage of creating this fractal, the subdivision continues into smaller and smaller equilateral triangles, where the "center triangle" is <u>removed</u> from all existing triangles. (Only count the "dark" triangles, not the triangular spaces left by removing triangles.)



Number of Triangles. *(10 points)*
Count the number of triangles at each stage 0 through 4 and then extend the pattern to predict how many triangles there will be at the nth stage.

STAGE	0	1	2	3	4	5	6	…	*n*
# OF TRIANGLES	1			27					

As *n* becomes larger and larger, going towards infinity, what happens to the number of triangles?

Area of Triangles. *(10 points)*
Let the area at stage 0 be equal to 1 (so the one large triangle we'll say has area 1). Find the total remaining (shaded) areas for the other stages and then extend this pattern to predict the area the remaining triangles cover at the n^{th} stage.

STAGE	0	1	2	3	4	5	6	…	*n*
AREA OF TRIANGLES	1	3/4		27/64					

As n becomes larger and larger, going towards infinity, what happens to the area?
So the SURPRISING result, which happens all the time when we look at fractal figures

is that as we continue building the fractal, the <u>number</u> of triangles tends to ____________

while the <u>area</u> of these triangles tends to ______________________!