Problem Set #3
(50 points)

Name: Score_____/50

Work on the following problems to turn in **Wednesday April 3**. Please make sure your work is clear, neat, and organized. A reminder: You may discuss these problems with each other, but your write-up and your submission must represent **your own work written independently of others**.

1. **Rates of Change**. [24 points]
   Use the idea of successively approximating the rate of change of a function at a given value using Leibniz' method where the slope is approximately equal to

$$\frac{f(x+h) - f(x)}{h}$$

   with smaller and smaller "h" values to find the derivative of $f(x) = x^3$ at the following points. Use the "h" values 0.1, 0.01, and 0.001 before determining the exact value of the rate of change.

1(a) Approximate the derivative of $f(x) = x^3$ at $x = 0$

Use $h = 0.1$

Use $h = 0.01$

Use $h = 0.001$

What do you believe the exact derivative of $f(x)$ at $x = 0$ is?

$f'(0) =$ _____

1(b) Approximate the derivative of $f(x) = x^3$ at $x = 1$

Use $h = 0.1$

Use $h = 0.01$

Use $h = 0.001$

What do you believe the exact derivative of $f(x)$ at $x = 1$ is?

$f'(1) =$ _____

1(c) It turns out that the exact derivative of $f(x) = x^3$ is $f'(x) = 3x^2$. Use this formula for the derivative of x^3 to find the derivative of $f(x) = x^3$ at $x = 0$ and $x = 1$ to confirm your work above

$x = 0$ $f'(0) =$

$x = 1$ $f'(1) =$

$x = a$ $f'(a) =$

[Note that you are now generalizing this to finding the derivative at an arbitrary point "***a.***"]

1(d) We now know the exact derivative of $f(x) = x^3$ is $f'(x) = 3x^2$. Suppose there existed an operation called anti-differentiation. What is the anti-derivative of $3x^2$?

2. **Integration Using Area Under a Curve**. [10 points]
Find the exact values of the following integrals using the idea of finding an area under a curve. Show your work.

a. $\int_0^1 (2x)\,dx$

b. $\int_0^2 (2x)\,dx$

c. $\int_0^a (2x)\,dx$

[Note that you are now generalizing this to an arbitrary ending point "***a.***"]

d. What is the antiderivative of 2*x*?

3. **Approximating the Values of Integrals**. [10 points]
Approximate the value of the following integral by splitting the interval into (a) 2 and then (b) 4 pieces and finding the area of rectangles that would overestimate the value of the integral.

$$\int_1^3 x^2\,dx$$

a. Approximation using 2 rectangles

b. Approximation using 4 rectangles

c. In class we were told that the integral of x^2 from a point to a second point was $x^3/3$ evaluated at the top endpoint minus $x^3/3$ evaluated at the bottom endpoint. Use this idea to calculate the exact value of the integral above.

4. **Anti-differentiation is Integration**! [6 points]
Think about the relationship between $x^3/3$ and x^2 and how these are used in this Problem Set. Explain in a few sentences (perhaps include equations) why we call finding integrals of functions "anti-differentiation." (This can be done based on something you might observe from your calculations above or based on information about certain derivatives that we discussed in class.)