## Problem Set \#2

( 50 points)

Name: $\qquad$
Work on the following problems to turn in Wednesday, March 20. Please make sure your work is clear, neat, and organized. A reminder: You may discuss these problems with each other, but your write-up and your submission must represent your own work written up independently of others.

1. Base $\mathbf{3}$ Addition Chart.

| + | 0 | 1 | 2 | 10 | 11 | 12 | 20 | 21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |  |  |
| 11 |  |  |  |  |  |  |  |  |
| 12 |  |  |  |  |  |  |  |  |
| 20 |  |  |  |  |  |  |  |  |
| 21 |  |  |  |  |  |  |  |  |

Describe any pattern(s) you see in this chart as a whole.
2. Some Arithmetic Problems in Base 5 and Base 7. Solve the following arithmetic problems twice, first in Base 5 and then again in Base 7. You do not need to show detailed work, but it may be helpful.

## (a) BASE 5.

| 34 | 213 | 22 |
| ---: | :--- | ---: |
| +22 |  |  |
| $-\quad$ five | -144 | $\underline{X_{\text {f } 14}}$ |

(b) BASE 7.

| 34 | 213 | 22 |
| ---: | :--- | ---: |
| +22 | -144 | $\underline{\text { x } 14}$ |
| $-\quad$ seven | $-\quad$ seven |  |

3. Propositional Logic and Venn Diagrams. Construct Venn Diagrams for each of the following two statements; this will show that they are logically equivalent.
(a) Use truth tables to show these statements are equivalent

$$
\mathbf{p}^{\wedge}(\neg \mathbf{q})
$$

$$
\neg((\neg \mathbf{p}) \vee \mathbf{q})
$$

(b) Use Venn Diagrams to show these expressions are equivalent
$\boldsymbol{P} \cap \overline{\boldsymbol{Q}}$

$$
\overline{(\overline{\boldsymbol{P}} \cup \boldsymbol{Q})}
$$

4. Propositional Logic and Truth Tables. Show that the following logical implications are NOT valid arguments by constructing the appropriate truth table and showing it does not give you a tautology.
(a) Fallacy of the inverse, i.e. "hypothetical denial"

$$
\left((\mathbf{p} \rightarrow \mathbf{q})^{\wedge} \neg \mathbf{p}\right) \rightarrow \neg \mathbf{q}
$$

(b) Fallacy of the converse, i.e. "consequential affirmation" $\left((\mathbf{p} \rightarrow \mathbf{q})^{\wedge} \mathbf{q}\right) \rightarrow \mathbf{p}$
5. Propositional Logic and English Sentences. Pick two statements pand $\mathbf{q}$ that should make sense in a "If p, then q" logical conclusion, such as "IF I have a PhD in Mathematics" THEN "I know how to add 3 digit numbers." Use your English sentence values for $\mathbf{p}$ and $\mathbf{q}$ and replicate the following syllogisms
(a) Fallacy of the inverse, i.e. "hypothetical denial" $\left((\mathbf{p} \rightarrow \mathbf{q})^{\wedge} \neg \mathbf{p}\right) \rightarrow \neg \mathbf{q}$
(b) Fallacy of the converse, i.e. "consequential affirmation" $\left((\mathbf{p} \rightarrow \mathbf{q})^{\wedge} \mathbf{q}\right) \rightarrow \mathbf{p}$
(c) Modus tollendo ponens or "Disjunctive syllogism" $\left((\mathbf{p} \mathbf{v} \mathbf{q})^{\wedge} \neg \mathbf{p}\right) \rightarrow \mathbf{q}$

