

**HW Set #1**  
(40 points)

Name: \_\_\_\_\_

Collaborators: \_\_\_\_\_

Work on the following problems to turn in **Monday February 5**. Please make sure your work is clear, neat, and organized. A reminder: You may discuss these problems with each other, but your write-up and your submission must represent **your own work written up independently of others**. Each problem is worth 5 points.

1. **Induction.** Using Induction, prove Gauss' formula (see "Gauss: The Child Genius" Handout) for the sum of the first  $n$  integers.

2. **Using Gauss' Formula.** Using Gauss' formula for the sum of the first  $n$  integers (see #1), find the sum of the numbers from 1 to 226.

Explain how you would use Gauss' formula to find the sum of the numbers from 17 to 226 (remember that Gauss' formula always starts adding at 1...so what do you do if you need to start summing at 17?). Show your work in finding this sum.

3. **Poison.** Explain in detail, using modular arithmetic, and in complete sentences, the winning strategy for a variation to the basic game of Poison. In this game, two players take turns picking 1, 2 or 3 chips and the last chip is “poison.” Who will win if there are 2673 chips on the board (and both players know the winning strategy)?

4. **Multiplication Algorithms.** Calculate  $27 \times 68$  by (1) the method of doubling, (2) the method of doubling switching the role of the two numbers from 1 above, and (3) the “grid” method we did in class.

5. **Some Details of Modular Arithmetic.** For  $a$  and  $b$  positive integers (actually it holds for all, including negative, integers)  $a \equiv b \pmod{n}$  if  $a$  and  $b$  have the same remainder when divided by  $n$ . **Give 5 different numbers that are equivalent to  $3 \pmod{4}$ .** Explain briefly how you went about generating these numbers. What can you say about the difference between any two of these numbers you wrote down (i.e., if you subtract any two of these numbers you listed, you'll get a number – what can you say about all these differences)?

6. **Application of Modular Arithmetic.** What are the last two digits of  $7^{2015}$ ?  
(HINT: Use the technique that was given on Worksheet #3 that involves looking at  $7^n \bmod 100$  will give the tens and ones digits of  $7^n$  for any value of  $n$ .)

Show all your work.

7. **Perfect Numbers.** Show your work in verifying that 28 and 496 are perfect numbers.

8. **Bank Codes.** Checks from any U.S. bank also use check digits to verify legitimate bank identification numbers, using mod 10 arithmetic. The first eight digits identify the bank, with the ninth being the check digit. The scheme here is a bit messier than the UPC or ISBN (leading actually to fewer undetectable errors, which is important to the financial markets):

$$7d_1 + 3d_2 + 9d_3 + 7d_4 + 3d_5 + 9d_6 + 7d_7 + 3d_8 + 9d_9 \equiv 0 \pmod{10}$$

If 12200024 is Wells Fargo's bank code, what must their check digit be? Explain clearly why this is (and has to be) a unique check digit (it must have something to do with the fact that you will multiply this digit by the number 9)?