An integral quadratic form is an expression of the form

\[ c_1w^2 + c_2x^2 + c_3y^2 + c_4z^2 \]

that can be denoted \((c_0, c_1, c_2, c_3)\) where \(w, x, y\) and \(z\) are ANY integers and the coefficients \(c_0, c_1, c_2\) and \(c_3\) are known (fixed) integers.

The Fifteen Theorem (Conway) says:

If there exists a \((c_0, c_1, c_2, c_3)\) quadratic form that will represent all the numbers from 1 to 15 inclusive then it can represent any other positive integer.

The Four Square Theorem (Lagrange) says:

The quadratic form \((1,1,1,1)\) can represent every positive integer.

1. Show that Ramanujan’s exceptional quadratic form given by \((1,2,5,5)\) DOES NOT represent all the positive integers from 1 to 15! (Provide examples of \(w, x, y\) and \(z\).)

2. Which of the integers between 1 and 15 does Ramanujan’s exceptional quadratic form miss?
3. Show that the quadratic form given by (1,2,4,5) DOES represent all positive integers from 1 to 15. (Provide examples of $w,x,y$ and $z$.)

4. Which of these quadratic forms, $p = w^2 + 2x^2 + 4y^2 + 5z^2$ or $q = w^2 + 2x^2 + 5y^2 + 5z^2$ is guaranteed to be able to represent any positive integer? EXPLAIN YOUR ANSWER!