These questions concern the Island of Knights and Knaves. Each person who lives on this island is either a Knight or a Knave. Every statement made by a Knight is true, while every statement made by a Knave is false.

Which type made the following statements, a Knight or a Knave? Explain the reasons for your answers.

1. \(1 + 1 = 2\).

   KNIGHT because it’s true.

2. \(2 \cdot 2 = 5\).

   KNAVE because it’s false.

3. I am a Knight.

   BOTH: KNIGHT because it’s true, KNAVE because he lies.

4. I am a Knave.

   NEITHER because the Knight wouldn’t tell a lie and the KNAVE wouldn’t be able to tell the truth.

5. My brother and I are both Knaves. (Which type is the speaker, which type is the speaker’s brother?)

   Can’t be a knight because he wouldn’t say this. So speaker is KNAVE lying so the brother is a KNIGHT.

6. I am a knave but my sister is not. (Which type is the speaker, which type is the speaker’s sister?)

   Knight can’t say he’s a Knaves so the speaker is a KNAVE, lying, so sister is KNAVE too.

7. Of my sister and I, at least one of us is a Knave. (Which type is the speaker, which type is the speaker’s sister?)

   If speaker were a KNAVE he wouldn’t say that at least one of them is a KNAVE because that would be true, so the speaker is KNIGHT, telling the truth, which makes the sister a KNAVE.

8. I am a Knav or there is gold on the island. (Which type is the speaker, and is there gold on the island?)

   Since is this an OR statement, the speaker couldn’t be a Knave lying because for an OR to be false both statements need to be false and if it was a Knave speaking the first part would be
true. For the statement to be true only one of the statement needs to be true so the speaker is KNIGHT telling the truth. And there is gold.

9. In this example, there are three people on the island: A, B, and C.
   A says: “All of us are knaves.”
   B says: “Exactly one of us is a knight.”
   (What are A, B, and C?)

   A couldn’t be a KNIGHT saying such a thing. So A is a Knave lying, so one of B or C is a Knight. If B were a Knave and C a Knight that would mean B was telling the truth which is a contradiction to his being a Knave. So B is a KNIGHT telling the truth making C a KNAVE.

10. In this example, there are three people on the island: A, B, and C.
    A says: “B and C are of the same type.”
    Someone asks C, “Are A and B of the same type?”
    (What does C answer?)

    Say A is Knight. So B and C are the same type. If both are Knights telling the truth then C says YES. If they are both Knaves lying so C lies and still says YES. If A is Knave so he lies and B and C are not the same type. If C Night and B Knave, C still tells the truth and says YES. If C Knave and B Knight, C lies about A and B and still says YES. So no matter what the answer is YES.

11. In this example, there are three people on the island: A, B, and C.
    A says: “B is a knave.”
    B says: “A and C are the same type.”
    (How many of the three are knights?)

    If A is a Knight, then B is a Knave, lying. So A and C are not the same. So C is a Knave and we have 1 KNIGHT, 2 KNAVES. If A is a Knave, lying, so B is Knight who tells the truth so C is Knave. We still have 1 KNIGHT, 2 KNAVES.