Problem Set #4
(70 points)

Name: ___________________________ Score: _______/70

Work on the following problems to turn in Monday November 23. Please make sure your work is clear, neat, and organized. A reminder: You may discuss these problems with each other, but your write-up and your submission must represent your own work written independently of others.

1. (10 points) Symmetries of the Square. On this page (use the back page if necessary), list out the eight (8) symmetries of the square. Begin with the following figure:

(i) Write a description (e.g., clockwise rotation of 63 degrees about the center) of each symmetry, then (ii) draw what the square above would be transformed to under that symmetry. Clearly number the eight symmetries and make sure I can tell which figure goes with which written description.
2. **(20 points) Dual Platonic Solids.** Explain clearly in a short paragraph what a cube is and what an octahedron is, and also explain completely why they are considered “dual” Platonic solids. Include visual representations (i.e. drawings of a cube and an octahedron). Also explain why one of the platonic solids is considered a “dual” Platonic solid with itself.
3. **(40 points)** **An Exploration of Fractal Geometry.** While we focused on 2-, 3-, and 4-dimensional spaces and objects in class, there is also a notion of *fractional* dimensions! This idea comes into play in the study of what are called “fractals” The purpose of this set of problems is to let you explore the idea of fractals, although we won’t get you to the idea of dimension here.

Some basic information on Fractal Geometry:
- In 1961 Benoit Mandelbrot really began the study of fractals in earnest.
- A fractal is a geometric figure that consists of an identical “motif” repeating itself over and over again, but on an ever-reduced scale.
- Fractals can be seen in nature – from the jagged edges of a coastline, to the intricate patterns of a snowflake, to the complex formation of fern leaves.

**Example A. Tree Fractal. (10 points)** The motif here is an inverted T. I’ve shown the first three stages of the construction of this fractal. Draw in the next stage.

![Tree Fractal Diagram](image)

Stage 1 Stage 2

Stage 3 Stage 4

Those of you interested in the NCAA basketball tournament, might see this as part of the brackets for a single-elimination tournament!
Example B. (10 points) Variation on the Tree Fractal. Here we draw the next stage by growing two branches, half as long as the previous branch, off the endpoint of each branch from the previous stage. We do so at 120-degree angles from the branch. Stage 1 (1 branch of length 16) and Stage 2 (2 branches of length 8) are drawn in for you. Draw in the details for the next three stages (with branches of length 4, 2, and 1 respectively).
**Example C. (20 points) Sierpinski Triangle.** Three of the first five stages (starting at stage 0) are drawn below. Fill in the two missing stages with careful drawings. In each subsequent stage of creating this fractal, the subdivision continues into smaller and smaller equilateral triangles, where the “center triangle” is removed from all existing triangles. (Only count the “dark” triangles, not the triangular spaces left by removing triangles.)

Number of Triangles. *(10 points)*
Count the number of triangles at each stage 0 through 4 and then extend the pattern to predict how many triangles there will be at the n
\(^{th}\) stage.

<table>
<thead>
<tr>
<th>STAGE</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>…</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td># OF TRIANGLES</td>
<td>1</td>
<td>27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
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As n becomes larger and larger, going towards infinity, what happens to the number of triangles?

Area of Triangles. *(10 points)*
Let the area at stage 0 be equal to 1 (so the one large triangle we’ll say has area 1). Find the total remaining (shaded) areas for the other stages and then extend this pattern to predict the area the remaining triangles cover at the n
\(^{th}\) stage.

<table>
<thead>
<tr>
<th>STAGE</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>…</th>
<th>n</th>
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<tbody>
<tr>
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<td>3/4</td>
<td>27/64</td>
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<td></td>
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As n becomes larger and larger, going towards infinity, what happens to the area? So the SURPRISING result, which happens all the time when we look at fractal figures is that as we continue building the fractal, the number of triangles tends to __________ while the area of these triangles tends to ________________!