

Nearly all of these either appeared on your HW or Problem of the Day.

These questions concern the Island of Knights and Knaves. Each person who lives on this island is either a Knight or a Knave. Every statement made by a Knight is true, while every statement made by a Knave is false.

Which type made the following statements, a Knight or a Knave? Explain the reasons for your answers.

1. $1 + 1 = 2$.

KNIGHT because it's true.

2. $2 \cdot 2 = 5$.

KNAVE because it's false.

3. I am a Knight.

BOTH: KNIGHT because it's true, **KNAVE** because he lies.

4. I am a Knave.

NEITHER because the Knight wouldn't tell a lie and the **KNAVE** wouldn't be able to tell the truth.

5. My brother and I are both Knaves. (Which type is the speaker, which type is the speaker's brother?)

Can't be a knight because he wouldn't say this. So speaker is KNAVE lying so the brother is a KNIGHT.

6. I am a knave but my sister is not. (Which type is the speaker, which type is the speaker's sister?)

Think of this as the compound statement: I am a knave AND my sister is not a knave. This compound statement is only TRUE when both parts are TRUE. The speaker can't be a Knight, because the first part would be false, which makes the entire statement FALSE. The speaker is a KNAVE, lying, so the sister is a KNAVE too, and the compound statement is FALSE.

7. Of my sister and I, at least one of us is a Knave. (Which type is the speaker, which type is the speaker's sister?)

If speaker were a KNAVE he wouldn't say that at least one of them is a KNAVE because that would be true, so the speaker is KNIGHT, telling the truth, which makes the sister a KNAVE.

8. I am a Knave or there is gold on the island. (Which type is the speaker, and is there gold on the island?)

Since is this IS an OR statement, the speaker can't be a Knave because the first part of thstatement would be TRUE and thus the entire statement would be a truthful one, which can not be spoken by a Knave. So the speaker is KNIGHT, and the compound statement must be TRUE. So there is gold and the speaker is a Knight.

9. In this example, there are two people on the island: A and B.

A says: "I am a knave, but B isn't." (What are A and B?)

Think of this as the compound statement: I am a Knave and B is not a Knave. There's no way the speaker can be a Knight, since the first part of the statement is false. So A must be a KNAVE lying about B which makes B a KNAVE as well.

10. In this example, there are three people on the island: A, B, and C.

A says: "All of us are knaves."

B says: "Exactly one of us is a knight."

(What are A, B, and C?)

A couldn't be a KNIGHT saying such a thing. So A is a Knave lying, so one of B or C is a Knight. If B were a Knave and C a Knight that would mean B was telling the truth which is a contradiction to his being a Knave. So B is a KNIGHT telling the truth making C a KNAVE.

11. In this example, there are three people on the island: A, B, and C.

A says: "B and C are of the same type."

Someone asks C, "Are A and B of the same type?"

(What does C answer?)

Say A is a Knight. So B and C are the same type. If both are Knights telling the truth then C says YES. If they are both Knaves lying so C lies and still says YES. If A is Knave so he lies and B and C are not the same type. If C is a Knight and B is a Knave, C still tells the truth and says YES. If C is a Knave and B is a Knight, C lies about A and B and still says YES. So no matter what the answer is YES.

12. In this example, there are three people on the island: A, B, and C.

A says: "B is a knave."

B says: "A and C are the same type."

(How many of the three are knights?)

If A is a Knight, then B is a Knave, lying. So A and C are not the same. So C is a Knave and we have 1 KNIGHT, 2 KNAVES. If A is Knave, lying, so B is Knight who tells the truth so C is Knave. We still have 1 KNIGHT, 2 KNAVES.