Triangular Numbers

The numbers 1, 3, 6, ... are called the first three triangular numbers since they may be represented by triangular patterns of dots as below:

- Draw pictures for a few more triangular numbers on the back.

- Make a table of the first 10 triangular numbers and, for any \( n \), give a formula for the \( n \)-th triangular number. Call the \( n \)th triangular number \( T_n \). To prove your formula, try to give a picture with dots which illustrates your result. (Notice that what you have actually found is a formula for the sum of the first \( n \) integers.)

- What is the sum of any two consecutive triangular numbers? That is find a formula for \( T_n + T_{n+1} \). Prove your answer is correct, using algebra and your result from the result above. Now draw a picture with dots to illustrate this result.

- Prove that if \( T \) is a triangular number, then so is \( 9T + 1 \).

- Explain why each number in this sequence is a triangular number:

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1, 1 + 9, 1 + 9 + 81, ..., 1 + 9 + 9^2 + ... + 9^k, ...
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