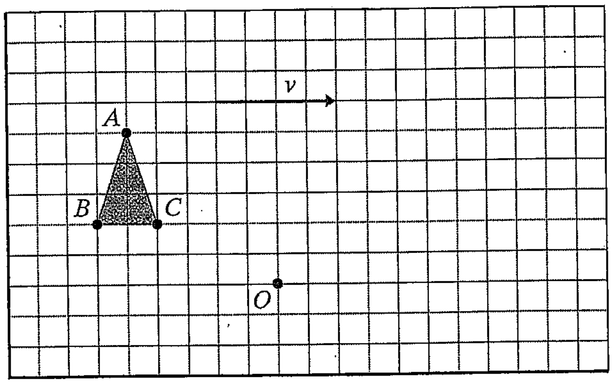
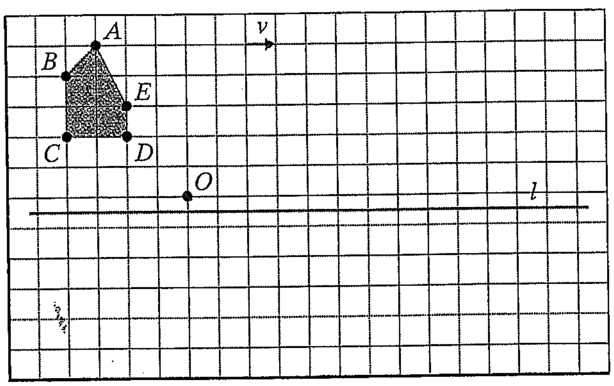
68. Suppose $\mathcal{M}$ is a translation by the vector v and $\mathcal{N}$ is a 90° clockwise rotation with rotocenter O, as shown in the following figure.



(a) Find the image of triangle ABC under the product of $\mathcal{M}$ and $\mathcal{N}$.

(b) Explain why the product of $\mathcal{M}$ and $\mathcal{N}$ is a rotation, and find the rotocenter and angle of the rotation.

69. Suppose $\mathcal{M}$ is a 90° clockwise rotation with rotocenter O, and $\mathcal{N}$ is a glide reflection with axis l and vector v, as shown in the following figure.



(a) Find the image of the shaded figure under the product of $\mathcal{M}$ and $\mathcal{N}$.

(b) Explain why the product of $\mathcal{M}$ and $\mathcal{N}$ is a reflection, and find the axis of the reflection.

70. Using copies of the symbol ♥ (and rotated versions of it), construct border patterns of symmetry type

(a) mg.

(b) $m1$.

(c) $1m$.

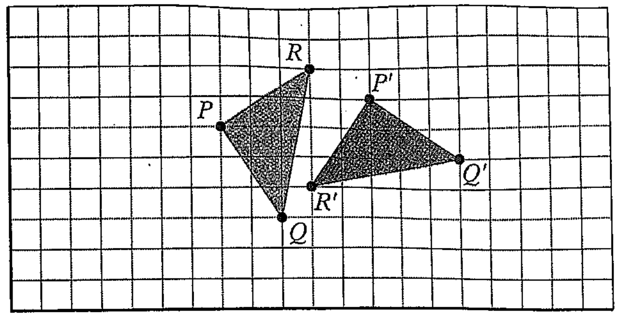
(d) 11.

(e) 12.

(f) 1g.

(g) mm.

71. A rigid motion $\mathcal{M}$ moves the triangle PQR into the triangle $P'Q'R'$ as shown in the figure. Explain why the rigid motion $\mathcal{M}$ must be a glide reflection.



72. A *palindrome* is a word that is the same when read forward or backward. MOM is a palindrome and so is ANNA. (For simplicity, we will assume all letters are capitals.)

(a) Explain why if a word has vertical reflection symmetry, then it must be a palindrome.

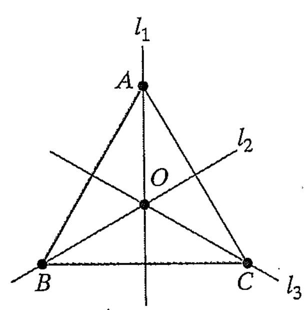
(b) Give an example of a palindrome (other than ANNA) that doesn't have vertical reflection symmetry.

(c) If a palindrome has vertical reflection symmetry, what can you say about the symmetries of the individual letters in the word?

(d) Find a palindrome with 180° rotational symmetry.

RUNNING

73. Let the six symmetries of the equilateral triangle ABC shown in the figure be denoted as follows: r_1: reflection with axis l_1, r_2: reflection with axis l_2, r_3: reflection with axis l_3, R_1: 120° clockwise rotation with rotocenter O, R_2: 240° clockwise rotation with rotocenter O, I: the identity symmetry.



Complete the following table by entering, in each row and column of the table, the symmetry that results when applying first the symmetry in the row followed by the symmetry in the column. (For example, the entry in row r_1 column r_2 is R_1 because the reflection r_1 followed by the reflection r_2 equals the rotation R_1.)