1. (a) Let $X_1 = \mathbb{R}$, $T_1 = \{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\mathbb{R}, \emptyset\}$. Prove that $T_1$ is a topology.

(b) Let $(X_2, T_2)$ be $\mathbb{R}$ with the standard topology (i.e., the topology induced by the Euclidean metric). Let $f : X_1 \to X_2$ and $g : X_2 \to X_1$ be given by $f(x) = x$ and $g(x) = x$. Is $f$ continuous? Is $g$ continuous? Prove your answers.

2. For $i = 1, 2$, let $(X_i, T_i)$ be a topological space.

(a) Show that if $T_1$ is the discrete topology, then every function $f : X_1 \to X_2$ is continuous.

(b) Show that if $T_2$ is the indiscrete topology, then every function $f : X_1 \to X_2$ is continuous.

3. For $i = 1, 2, 3$, let $(X_i, T_i)$ be a topological space. Let $f : X_1 \to X_2$ and $g : X_2 \to X_3$ be continuous maps. Prove that their composition $g \circ f : X_1 \to X_3$ is continuous.