1. Let \( f : \mathbb{R} \to \mathbb{R} \) be given by: 
\[
 f(x) = \begin{cases} 
 1/2 & \text{if } x < 0 \\
 1/3 & \text{if } x \geq 0
\end{cases}
\]. Prove that \( f \) is not continuous at 0.

2. In the following, just find a map each problem asks for, without proving continuity, injectivity, or surjectivity. Each of the following sets is assumed to come with the standard Euclidean metric.

   (a) Let \( M_1 \subset \mathbb{R}^2 \) be the closed unit disk (i.e., \( M_1 = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\} \)). Let \( M_2 \subset \mathbb{R}^2 \) be the closed disk of radius 2 centered at the origin. Find a continuous bijection (one-to-one and onto map) \( f : M_1 \to M_2 \).

   (b) Let \( M_3 \subset \mathbb{R}^2 \) be the closed disk of radius 1 centered at the point \((3,4)\). Find a continuous bijection \( f : M_1 \to M_3 \).

   (c) Let \( M_4 \subset \mathbb{R}^2 \) be the closed disk of radius 2 centered at the point \((3,4)\). Find a continuous bijection \( f : M_1 \to M_4 \).

3. Suppose \( M_1 = (X_1,d_1) \) and \( M_2 = (X_2,d_2) \) are metric spaces. Pick a point \( b \in X_2 \), and let \( f : X_1 \to X_2 \) be the constant map \( f(x) = b \), \( \forall x \in X_1 \). Show that \( f \) is continuous on \( X_1 \).

4. Suppose \( M_1 = (X_1,d_1) \) and \( M_2 = (X_2,d_2) \) are metric spaces, and suppose \( f : X_1 \to X_2 \) is a continuous function. Prove that \( \forall a \in X_1 \) and \( \forall \varepsilon > 0 \), \( \exists \delta > 0 \) such that the ball of radius \( \delta \) around \( a \) is mapped under \( f \) to inside the ball of radius \( \varepsilon \) around \( f(a) \); i.e., \( f(B_\delta(a)) \subseteq B_\varepsilon(f(a)) \).

5. Suppose \( M_1 = (X_1,d_1) \) and \( M_2 = (X_2,d_2) \) are metric spaces, and suppose \( f : X_1 \to X_2 \) is a continuous function. Prove that the preimage of any open set in \( M_2 \) is an open set in \( M_1 \); i.e., if \( A_2 \subset X_2 \) is open, then \( A_1 = f^{-1}(A_2) \subset X_1 \) is also open.