1. Prove that a space \( X \) is connected iff it contains no nonempty, proper subset which is both open and closed. (To say \( A \) is a proper subset of \( X \) means \( A \subset X \) but \( A \neq X \).)

2. True or false: If \( A \) and \( B \) are not disjoint, and each is a connected subspace of a topological space \( X \), then \( A \cap B \) is connected. Prove your answer.

3. True or false: If \( A \) and \( B \) are not disjoint, and each is a connected subspace of a topological space \( X \), then \( A \cup B \) is connected. Prove your answer.

4. (a) Prove the following theorem:
   Theorem: The continuous image of a connected set is connected; i.e, if \( f : X \to Y \) is a continuous map between topological spaces, and if \( X \) is connected, then \( f(X) \) is connected.

   (b) Prove the following corollary:
   Corollary: If \( X \) is connected, and \( Y \) is homeomorphic to \( X \), then \( Y \) is connected.

Extra Credit Problems

5. Prove the following theorem: \( A \subset \mathbb{R} \) is connected iff \( A \) is an interval (open, closed, or half open; infinite or half-infinite).