1. Suppose $(X_1, d_1)$ and $(X_2, d_2)$ are metric spaces, and suppose $f : X_1 \to X_2$ is a function such that the preimage of every open set is open, i.e., for every open set $A_2 \subset X_2$, $f^{-1}(A_2)$ is open in $X_1$. Prove that $f$ is continuous according to the definition of continuity for metric spaces.

2. Find all topologies on the set $X = \{a, b, c\}$. Just list the different topologies, without proving that they really are topologies. In your list, identify the discrete and the indiscrete topologies.

3. Let $X = \mathbb{R}$, $T = \{[a, b] \mid a, b \in \mathbb{R}\} \cup \{\mathbb{R}, \emptyset\} \cup \{(a, \infty) \mid a \in \mathbb{R}\} \cup \{(-\infty, b) \mid b \in \mathbb{R}\}$. Is $T$ a topology? Prove your answer.

4. Let $(X, d)$ be a metric space. Prove that for every point $p \in X$, the set $\{p\}$ is closed in $X$.

Extra Credit Problems

5. Find a homeomorphism between the Cantor set $C$ (see previous homework for definition) and its left half, i.e., $C \cap [0, 1/3]$. 