Let $FA = \{ B \subseteq A \mid \exists \text{new}(n \approx B) \}$.

Show if $A$ is infinite, $A \approx FA$.

**Proof**

1. Let $X_n = \{ f : n \to A \mid f \text{ is 1-1} \}$.
2. Let $F_n A = \{ B \subseteq A \mid n \approx B \}$.
3. $X_n \approx F_n A$, b/c:
   - Let $\varphi : X_n \to F_n A$ be given by $\varphi(f) = \text{ran } f$. Then $\varphi$ is a bijection.
4. Let $Y_n = \{ f : n \to A \} = ^n A$
   - Then $X_n \subseteq Y_n$, so
   - $\text{Card } X_n \leq \text{Card } Y_n$.
5. $\text{Card} (^n A) = (\text{Card } A)^n$ by def of exponentiation,
   - so $\text{Card } X_n \leq (\text{Card } A)^n$, by 4,
   - $\leq \text{Card } A$ by Absorption Law (since $A$ is infinite).
6. $FA = \bigcup_{\text{new}} F_n A$, so

$\text{Card } FA = \text{Card} \left( \bigcup_{\text{new}} F_n A \right)$

$\leq \omega \cdot \text{Card } A$ by 5, & by Problem 26 P.161

$= \text{Card } A$, by Absorption Law.

So $FA \leq A$.

Also, $A \leq FA$ b/c

$g : A \to FA$ where $g(a) = \{ a \}$ is 1-1.

So, by 6 & 7,

$A \approx FA$. \(\square\)