(a) $A \neq \emptyset$; $A \in A \Rightarrow A$ is a trans reln. Show
$\cap A$ is a trans reln.

Pf. 1. Spse $<x, y> \in \cap A$
   $\land <y, z> \in \cap A$

2. Let $A$ be an arbitrary element of $\mathfrak{A}$.

3. $<x, y> \in A \land <y, z> \in A$
   by 1, 2 & def of $\cap$.

4. $<x, z> \in A$, by 3 &
   def of trans, b/c $A$ is trans.

5. $<x, z> \in \cap A$ by 4
   & 2 (b/c $A$ is arbitrary
   elem of $\mathfrak{A}$.)

6. $\cap A$ is trans by 1 & 5
   & def of trans. $\Box$

(b) False: Let $A_1 = \{<a, b>\}$,
 $A_2 = \{<b, c>\}$, where
 $a, b, c$ are distinct sets.
Then $A_1 \cup A_2$ are trans,
but $A_1 \cup A_2$ is not. $\Box$
"⇒": Suppose R is symm & trans.  
WTS: R = R^−1 o R. 
1. Suppose <x, y> ε R 
2. <y, x> ε R, since R symm.
3. yRx ∨ xRy, by 1 & 2.
4. yRy, by 3 & R trans.
5. yR^−1 y, by def of R^−1 
6. yR^−1 y ∨ xRy, by 1 & 5
7. <y, x> ε R^−1 o R, by 6 & def of compos. 
8. <x, y> ε R ⇒ <y, x> ε R^−1 o R, by 1 & 7.
9. Now suppose <x, y> ε R^−1 o R, 
10. ∃z (xRz ∨ zR^−1 y), by def of compos.
11. xRz ∨ yRz, by 10 & def of R^−1 
12. xRz ∨ zRy, by R symm 
13. xRy, by 12 & R trans.
14. <x, y> ε R^−1 o R ⇒ <y, x> ε R, by 9 & 13.

⇐": Suppose R = R^−1 o R (hypothesis) 
(i) WTS: R is symm, 
1. Suppose <x, y> ε R. 
2. <x, y> ε R^−1 o R, by hypoth & 1. 
3. ∃z (xRz ∨ zR^−1 y), by 2 & def of compos. 
4. zR^−1 x ∨ yRz, by 3 & def of R^−1. 
5. <y, x> ε R^−1 o R, def of compos & 4.
6. <y, x> ε R, by hypoth & 5.
7. R is symm by 1 & 6.
(ii) WTS: R is trans. 
1. Suppose xRy & yRz. 
2. xRy & zRy, since R symm. 
3. xRy & yR^−1 z, by 2 & def R^−1. 
4. <x, z> ε R^−1 o R, by 3 & def of compos. 
5. <x, z> ε R, by 4 & hypoth. 
6. R trans, by 1 & 5. □