All of the following problems concern Statement Calculus (not Predicate Calculus).

1. Is each of the following True or False? Prove your answers.

   (a) Let $\Gamma$ be a (possibly infinite) set of formulas. If $\Gamma$ is not satisfiable, then it contains a formula $A$ that’s not satisfiable.

   (b) Let $A$ be a formula and $\Gamma$ and $\Delta$ (possibly infinite) sets of formulas. If $\Gamma \vdash A$ and, for every formula $B \in \Gamma$, $\Delta \vdash B$, then $\Delta \vdash A$. 
2. (a) State the Strong Completeness and the Strong Soundness Theorems. Then use these theorems to prove the following:

Let $B$ be a formula, and $\Gamma$ a set of formulas. If $\Gamma \models B$, then for some finite subset $\Delta$ of $\Gamma$, $\Delta \models B$.

(b) State the Compactness Theorem. Then use the Compactness Theorem (but not the Strong Completeness and the Strong Soundness Theorems) to prove the following:

Let $B$ be a formula, and $\Gamma$ a set of formulas. If $\Gamma \models B$, then for some finite subset $\Delta$ of $\Gamma$, $\Delta \models B$. 