Some of the following is a review from Calculus. You should become comfortable with them.

Definition: Let’s say \( f(n) \) dominates \( g(n) \) if \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = \pm\infty \) (or, equivalently, \( \lim_{n \to \infty} \frac{g(n)}{f(n)} = 0 \)).

1. Assume \( f(n) \) and \( g(n) \) both tend to infinity as \( n \) tends to infinity. Is there any relation between “\( f \) dominates \( g \)”, “\( g \) dominates \( f \)”, and “\( f \) is \( O(g) \)”\)? (For example, are two of them equivalent, or does one imply another?) Prove your answer.

2. Prove that every polynomial is dominated by \( n^{\log n} \).

3. Sort the following functions in order of dominance.

   \[ f(n) = n^{\log n}, \quad g(n) = 2^n, \quad h(n) = n^{\sqrt{n}}, \quad j(n) = n^{0.1n}, \quad k(n) = 1.1^n, \quad l(n) = n^n, \quad m(n) = n! \]

Hints: 1. Use L’Hopital’s rule. 2. Use Stirling’s approximation: \( n! \approx n^n \sqrt{2\pi n}/e^n \), where \( r(n) \approx s(n) \) means as \( n \) tends to infinity, \( r(n)/s(n) \) tends to 1.