Problem #1

A marketing survey indicates that 60% of the population owns an automobile, 30% owns a house, and 20% owns both an automobile and a house.

Calculate the probability that a person chosen at random owns an automobile or a house, but not both.

A. 0.4

B. 0.5

C. 0.6

D. 0.7

E 0.9

Problem #2

In a country with a large population, the number of persons, N, that are HIV positive at time t is given by:

$$N = 1000 \ln(t + 2), \; t \geq 0$$

Determine N at the time when the maximum rate of change in the number of persons that are HIV positive occurs.

A. 0

B. 250

C. 500

D. 693

E. 1000

Problem #3

Ten percent of a company's life insurance policyholders are smokers. The rest are non-smokers.

The probability of a non-smoker dying during the year is 0.01. The probability of a smoker dying during the year is 0.05.

The times of death for both smokers and non-smokers follow uniform distributions during the year.

Calculate the probability that the first policyholder to die during the year is a smoker.

A. 0.05

B. 0.20

C. 0.36

D. 0.56

E. 0.90

Problem #4

Let X and Y be random losses with the joint density function:

$$f(x,y) = e^{-(x+y)} \qquad \text{with } 0 < x < \infty \text{ and } 0 < y < \infty.$$

An insurance policy is written to cover the loss $X + Y$.

Calculate the probability that the loss is less than 1.

A. e^{-2}

B. e^{-1}

C. $1 - e^{-1}$

D. $1 - 2e^{-1}$

E. $1 - 2e^{-2}$

Problem #5

The rate of change of the population of a town in Pennsylvania at any time t is proportional to the population at time t. Four years ago, the population was 25,000. Now, the population is 36,000.

Calculate what the population will be six years from now.

A. 43,200

B. 52,500

C. 62,208

D. 77,760

E. 89,580

Problem #6

Calculate $\int_0^\infty \int_0^x (1+x^2+y^2)^{-2}\ dy\, dx$.

A. 0

B. $\frac{p}{16}$

C. $\frac{p}{8}$

D. $\frac{p}{4}$

E. p

Problem #7

As part of the underwriting process for insurance, each prospective policyholder is tested for high blood pressure.

Let X represent the number of persons tested until the first person with high blood pressure is found. The expected value of X is 12.5.

Calculate the probability that the sixth person tested is the first one with high blood pressure.

A. 0.000

B. 0.053

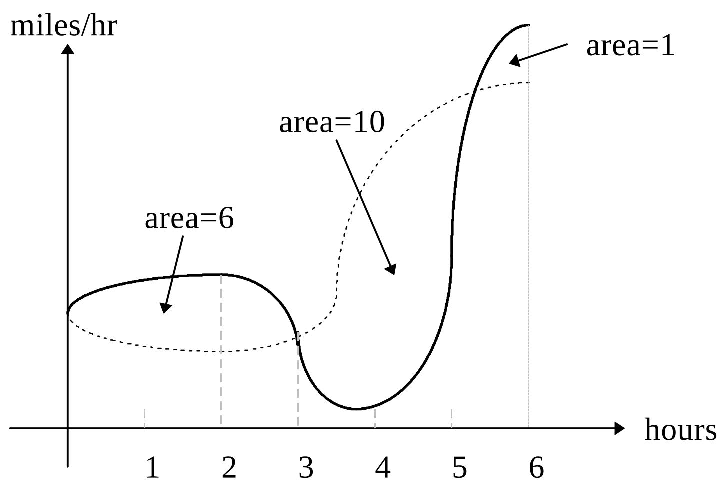
C. 0.080

D. 0.316

E. 0.394

Problem #8

At time $t = 0$, car A is five miles ahead of car B on a stretch of road. Both cars are traveling in the same direction. In the graph below, the velocity of A is represented by the solid curve and the velocity of B is represented by the dotted curve.



Determine the time(s), *t*, on the time interval (0, 6], at which car A is exactly five miles ahead of car B.

A. at $t = 2$.

B. at $t = 3$.

C. at some t, $3 < t < 5$, which cannot be determined precisely from the information given.

D. at $t = 3$ and at $t = 5$.

E. Car A is never exactly five miles ahead of Car B on (0,6].

Problem #9

The distribution of loss due to fire damage to a warehouse is:

Amount of Loss	Probability
$ 0	0.900
500	0.060
1,000	0.030
10,000	0.008
50,000	0.001
100,000	0.001

Given that a loss is greater than zero, calculate the expected amount of the loss.

A. $ 290

B. $ 322

C. $ 1,704

D. $ 2,900

E. $ 32,222

Problem #10

A manufacturer can invest a total of 30 in the production of two products. Product A costs twice as much to produce as Product B.

The utility to the manufacturer from producing the two products is given by the function:

$$G(x,y) = xy + 2x$$

where x is the number of units of Product A produced and y is the number of units of Product B produced.

Calculate the maximum value of $G(x,y)$.

A. 128

B. 140

C. 156

D. 240

E. 255

Problem #11

The risk manager at an amusement park has determined that the cost of accidents is a function of the number of people in the park. The cost is represented by the following function:

$C(x) = x^3 - 6x^2 + 15x$, where x is the number of people (in thousands) in the park.

The park self-insures this cost by including a charge of 0.01 per person in the price of every ticket to cover the cost of accidents

Calculate the number of people (in thousands) in the park that provides the greatest margin in the total amount collected from the insurance charge over the total cost of accidents

A. 0.47

B. 0.53

C. 2.00

D. 3.47

E. 3.53

Problem #12

An investor invests \$100. The value, I, of the investment at the end of one year is given by the equation:

$$I = 100\left(1+\frac{c}{n}\right)^{n}$$

where c is the nominal rate of interest and n is the number of interest compounding periods in one year.

Determine I if there are an infinite number of compounding periods in one year.

A. 100

B. $100ec$

C. $100e^{c}$

D. $100e^{\frac{1}{c}}$

E. ∞

Problem #13

The number of claims for an insurance policy has a binomial distribution with 2 trials and $p = 0.2$, where p is the probability of a claim. The claim size is random and independent of the number of claims with mean 1 and variance 2.

Calculate the variance of the total amount of claims paid under this policy.

A. 1.04

B. 1.12

C. 1.16

D. 1.20

E. 1.32

Problem #14

Workplace accidents are categorized in three groups: minor, moderate and severe. The probability that a given accident is minor is 0.5, that it is moderate is 0.4, and that it is severe is 0.1.

Two accidents occur independently in one month.

Calculate the probability that neither accident is severe and at most one is moderate.

A. 0.25

B. 0.40

C. 0.45

D. 0.56

E. 0.65

Problem #15

A life insurance company wants to issue one-year term life insurance contracts to two classes of independent lives, as shown below.

Class	Probability of Death	Benefit Amount	Number in Class
A	0.01	$200,000	500
B	0.05	$100,000	300

The company wants to collect an amount, in total, equal to the 95th percentile of the distribution of total claims.

The company will collect an amount from each life insured that is proportional to that life's expected claim. That is, the amount for life j with expected claim $E\left[X_j\right]$ would be $(1+q)E\left[X_j\right]$.

Calculate q .

A. 0.30

B. 0.32

C. 0.34

D. 0.36

E. 0.38

Problem #16

Micro Insurance Company issued insurance policies to 32 independent lives. For each policy, the probability of a claim is 1/6. The benefit amount given that there is a claim has probability density function

$$f(y) = \begin{cases} 2(1-y), & 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate the expected value of total benefits paid.

A. $\frac{16}{9}$

B. $\frac{8}{3}$

C. $\frac{32}{9}$

D. $\frac{16}{3}$

E. $\frac{32}{3}$

Problem #17

An actuary is reviewing a study she performed ten years ago on the size of claims made under homeowners insurance policies.

In her study, she concluded that the size of claims followed an exponential distribution and that the probability that a claim would be less than $1,000 was 0.250.

The actuary feels that the conclusions she reached in her study are still valid today with one exception: every claim made today would be twice the size of a similar claim made ten years ago as a result of inflation.

Calculate the probability that the size of a claim made today is less than $1,000.

A. 0.063

B. 0.125

C. 0.134

D. 0.163

E. 0.250

Problem #18

Let $F(x)$ represent the fraction of payroll earned by the highest paid fraction x of employees in a company (for example $F(0.2) = 0.5$ means that the highest paid 20% of workers earn 50% of the payroll).

Gini's index of inequality, G, is one way to measure how evenly payroll is distributed among all employees and is defined as follows:

$$G = 2 \int_0^1 \left| x - F(x) \right| dx .$$

In a certain company, the distribution of payroll is described by the density function:

$$f(x) = 3(1-x)^2 \text{ , for } 0 \le x \le 1$$

Calculate G for this company.

A. 0.0

B. 0.4

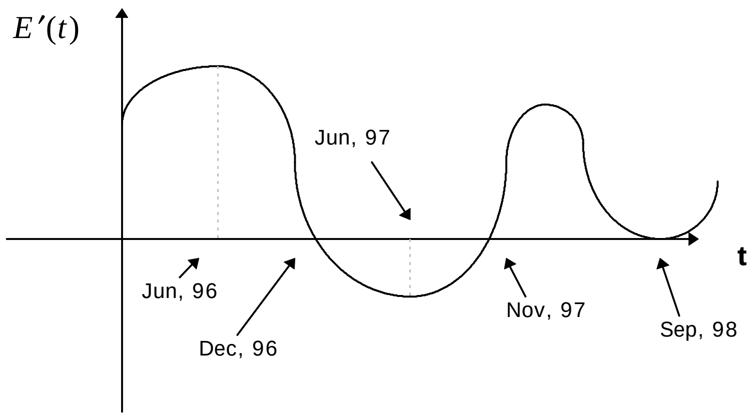
C. 0.5

D. 1.0

E. 1.5

Problem #19

According to classical economic theory, the business cycle peaks when employment reaches a maximum, relative to adjacent time periods. Employment, can be approximated as a function of time, t, by a differentiable function $E(t)$. The graph of $E'(t)$ is pictured below.



Which of the following points represents a peak in the business cycle?

A. Jun, 96

B. Dec, 96

C. Jun, 97

D. Nov, 97

E. Sep, 98

Problem #20

Let $f(x) = \int_{2}^{3x} \sin^2 t \, dt$.

Calculate $f''(x)$.

A. $\sin^2(3x)$

B. $3 \sin^2(3x)$

C. $2 \sin(3x) \cos(3x)$

D. $6 \sin(3x) \cos(3x)$

E. $18 \sin(3x) \cos(3x)$

Problem #21

An economist defines an "index of economic health," D, as follows:

$$D = E^2 (100 - I)$$

where: E is the percent of the working-age population that is employed and
I is the rate of inflation (expressed as a percent).

On June 30, 1996, employment is at 95% and is increasing at a rate of 2% per year and the rate of inflation is at 6% and is increasing at a rate of 3% per year

Calculate the rate of change of D on June 30, 1996.

A. -9,503 per year

B. -9,500 per year

C. 0 per year

D. 8,645 per year

E. 17,860 peryear

Problem #22

A dental insurance policy covers three procedures: orthodontics, fillings and extractions. During the lifetime of the policy, the probability that the policyholder needs:

- orthodontic work is 1/2
- orthodontic work or a filling is 2/3
- orthodontic work or an extraction is 3/4
- a filling and an extraction is 1/8

The event that the policyholder needs orthodontic work is independent of the need for either a filling or an extraction.

Calculate the probability that the policyholder will need either a filling or an extraction during the life of the policy.

A. 7/24

B. 3/8

C. 2/3

D. 17/24

E. 5/6

Problem #23

The value, v, of an appliance is based on the number of years since manufacture, m, as follows:

$$v(m) = e^{(7 - 0.2m)}$$

The warranty, w, on the appliance is defined as follows,

$$w(m) = \begin{cases} v(m) & 0 < m \leq 1 \\ 0.9v(m) & 1 < m \leq 7 \\ 0 & m > 7 \end{cases}$$

The probability of the appliance failing follows an exponential distribution with mean 10.

Calculate the expected value of the warranty.

A. 98.70

B. 109.66

C. 270.43

D. 298.18

E. 352.16

Problem #24

An automobile insurance company divides its policyholders into two groups: good drivers and bad drivers. For the good drivers, the amount of an average claim is $1,400, with a variance of 40,000. For the bad drivers, the amount of an average claim is $2,000, with a variance of 250,000. Sixty percent of the policyholders are classified as good drivers.

Calculate the variance of the amount of a claim for a policyholder.

A. 124,000

B. 145,000

C. 166,000

D. 210,400

E. 235,000

Problem #25

Let S be the region in the first quadrant of the xy-plane bounded by $y = \sqrt{x}$, $x - y = 2$ and the x-axis.

Calculate $\iint_S y \, dA$.

A. 13/12

B. 4/3

C. 9/4

D. 8/3

E. 10/3

Problem #26

Let X be a random variable with moment generating function

$$M(t) = \left(\frac{2+e^t}{3}\right)^9, \quad -\infty < t < \infty .$$

Calculate the variance of X.

A. 2

B. 3

C. 8

D. 9

E. 11

Problem #27

The annual number of claims filed under a block of disability income insurance policies has been constant over a ten year period, but the number of claims outstanding does exhibit seasonal fluctuations. The number of outstanding claims peaks around the first of the year, declines through the first two quarters of the year, reaches its lowest level around July 1, then climbs again to regain its peak level on January 1.

Which of the following functions best represents the number of outstanding claims, as a function of time, *t*, where *t* is measured in months and *t* = 0 on January 1, 1987.

A. $k \cos\left(p\, t/6\right)$ where k is a constant greater than zero

B. $k \cos\left(p\, t/12\right)$ where k is a constant greater than zero

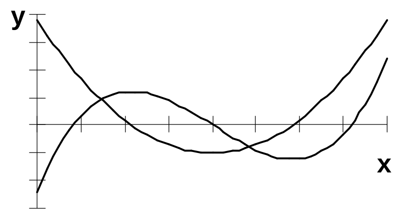
C. $k \cos\left(p\, t/12\right) + c$ where c and k are constants greater than zero

D. $k \cos\left(p\, t/12\right) + ct$ where c and k are constants greater than zero

E. $k \cos\left(p\, t/6\right) + ct$ where c and k are constants greater than zero

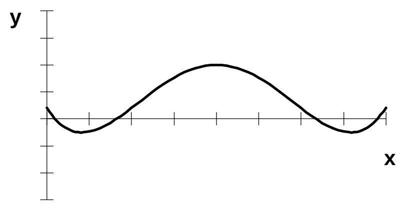
Problem #28

The graphs of the first and second derivatives of a function are shown below, but are not identified from one another.

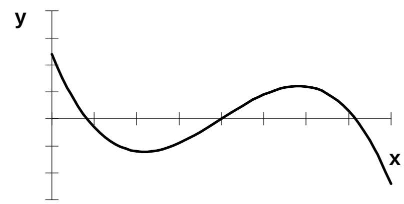


Which of the following could represent a graph of the function?

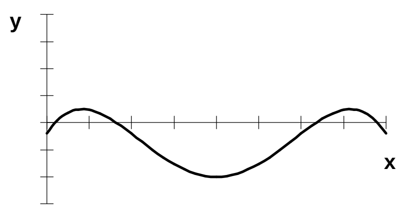
A.



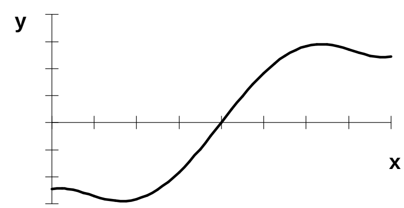
D.



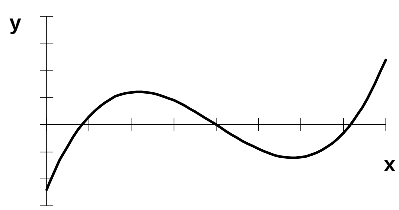
B.



E.



C.



Problem #29

Studies indicate that 10% of the population have a defective gene which makes them more susceptible to contracting communicable diseases.

An insurance company estimates that, for a person with the defective gene, the probability of *n* claims in a year on a medical insurance policy is given by a Poisson distribution with mean 0.6. For a person without a defect in the gene, the probability of *n* claims in a year is given by a Poisson distribution with mean 0.1.

The company does not know which of its policyholders have the defective gene and which do not, but does believe that its distribution of policyholders mirrors the population in general.

Calculate the expected number of claims this year for a policyholder who had one claim on his medical insurance policy last year.

A. 0.15

B. 0.18

C. 0.24

D. 0.30

E. 0.40

Problem #30

The amount of loss, in dollars, from a hurricane to a woodframe house is a function of the number of miles, *x*, the house is located from the coastline as follows:

$$f(x) = \sqrt[x]{(e^x + 3x)}\ , x > 0$$

Calculate the amount of loss, in dollars, from a hurricane to a woodframe house which is constructed as close as possible to the coastline.

A. 1

B. 3

C. 4

D. e^3

E. e^4

Problem #31

Let X and Y be random losses with joint density function,

$$f(x,y) = 2x \qquad \text{with } 0 < x < 1 \text{ and } 0 < y < 1.$$

An insurance policy is written to cover the loss $X+Y$. The policy has a deductible of 1.

Calculate the expected loss payment under the policy.

A. 1/4

B. 1/3

C. 1/2

D. 7/12

E. 5/6

Problem #32

Curve C_1 is represented parametrically by $x = t + 1$, $y = 2t^2$. Curve C_2 is represented parametrically by $x = 2t + 1$, $y = t^2 + 7$.

Determine all the points at which the curves intersect.

A. (3,8) only

B. (1,0) only

C. (3,8) and (-1,8) only

D. (1,0) and (1,7) only

E. The curves do not intersect anywhere

Problem #33

The number of clients a stockbroker has at the end of the year is equal to the number of new clients she is able to attract during the year plus the number of last year's clients she is able to retain.

Because servicing existing clients takes away from the time she can devote to attracting new ones, the stockbroker acquires fewer new clients when she has a lot of existing clients.

Let C_n represent the number of clients the stockbroker has at the end of year n, where:

$$C_n = \frac{2}{3} C_{n-1} + \frac{9000}{C_{n-1}^2}.$$

The stockbroker has five clients when she starts her business at year $n = 0$.

Calculate the number of clients she will have in the long run.

A. 3

B. 5

C. 10

D. 30

E. 363

Problem #34

Under an insurance policy, an insurer agrees to pay 100% of the actual loss incurred on the first accident in which the insured is involved during the year, up to a maximum payment of $1,000. The probability of the insured being in an accident during the year is 0.4. If an accident does occur, the amount of the loss, X, has a probability density function.

$$f(x) = \begin{cases} x(4-x)/9 & \text{for } 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

where x is measured in thousands of dollars.

Determine the expected amount of the claim an insurer would pay during the year, in thousands of dollars.

A. $13/270$

B. $13/108$

C. $101/270$

D. $101/108$

E. $151/108$

Problem #35

Let X and Y denote the remaining lifetimes of a husband and wife. Assume that the remaining lifetime of each person has an exponential distribution with mean l and that the remaining lifetimes are independent.

An insurance company offers two products to married couples:

One which pays when the first spouse dies, that is, at time $\min(X, Y)$; and

One which pays when the second spouse dies, that is, at time $\max(X, Y)$.

Calculate the covariance between the two payment times.

A. $l^2/6$

B. $l^2/4$

C. $l^2/3$

D. $l^2/2$

E. l^2

Problem #36

An index of consumer confidence fluctuates between -1 and 1. Over a two year period, beginning at time $t = 0$, the level of this index, c, is closely approximated by

$$c(t) = \frac{t\cos\left(t^2\right)}{2}, \text{ where } t \text{ is measured in years.}$$

Calculate the average value of the index over the two year period.

A. $-\frac{1}{4}\sin(4)$

B. 0

C. $\frac{1}{8}\sin(4)$

D. $\frac{1}{4}\sin(4)$

E. $\frac{1}{2}\sin(4)$

Problem #37

Let X and Y be random losses with joint density function:

$$f(x, y) = \begin{cases} 2(x + y), & 0 < x < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

An insurance policy is written to cover the loss X.

Calculate the expected value of Y.

A. 5/12

B. 1/2

C. 3/4

D. 1

E. 7/6

Problem #38

Isabelle N. Vest bought one share of stock issued by SloGro, Inc. The stock paid annual dividends. The first dividend Ms. Vest received was one dollar. Each subsequent dividend was five percent less than the previous one.

After receiving 40 dividend payments, Ms. Vest sold the stock.

Calculate the total amount of dividends Ms. Vest received.

A. $ 8.03

B. $17.43

C. $20.00

D. $32.10

E. $38.00

Problem #39

The loss amount, X, for a medical insurance policy has the following cumulative distribution function:

$$F(x) = \begin{cases} \frac{1}{9}\left(2x^2 - \frac{x^3}{3}\right), & 0 \le x \le 3, \\ 0, & \text{otherwise} \end{cases}$$

Calculate the mode of the distribution.

A. 2/3

B. 1

C. 3/2

D. 2

E. 3

Problem #40

A small commuter plane has 30 seats. The probability that any particular passenger will not show up for a flight is 0.10, independent of other passengers. The airline sells 32 tickets for the flight.

Calculate the probability that more passengers show up for the flight than there are seats available.

A. 0.0042

B. 0.0343

C. 0.0382

D. 0.1221

E. 0.1564

Problem #41

Problem	Key
1	B
2	D
3	C
4	D
5	C
6	C
7	B
8	C
9	D
10	A
11	E
12	C
13	B
14	E
15	D
16	A
17	C
18	C
19	B
20	E
21	D
22	D
23	D
24	D
25	D
26	A
27	C
28	A
29	C
30	E
31	A
32	C
33	D
34	C
35	B
36	C
37	C

Problem #42

38	B
39	D
40	E