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Which Door has the Cadillac?: Part II

by Andrew Vazsonyi, Feature Editor

The first part of this column was published in the December/January 1999 issue of *Decision Line*, available at http://www.reallifemath.com/vazs30_1.pdf. Since then, many things have changed. Publication of my memoir, *Which Door Has The Cadillac, Adventures of a Real-Life Mathematician* (2002), has triggered off many questions and doubts over the game-show problem in which a contestant is asked to choose which of three doors may hide a Cadillac—while the other two doors conceal a goat. (This is known as the Monte Hall problem from the game show "Let's Make a Deal." Out of curiosity I checked it on the Google browser and had an astounding 146,000 hits!) Surprisingly, if the contestant chooses a door, and then the emcee opens another door revealing a goat, the contestant improves her chances by picking the other remaining door. I was surprised that so many people are intrigued by this problem. They especially desire a "non-mathematical" explanation to it.

Back in 1999 I was not interested in a "non-mathematical" solution. I just developed my own probability tree to find the probability of winning in the case of switching. To begin drawing the tree, I assumed that the probability was 1/3 that the car was behind each of the doors. Exhibit A shows the part of the tree if the car happened to be behind Door #1. There are three cases: if I guess #1 (probability 1/3 * 1/3). The emcee opens #2 or #3, but what rule does he follow? I assume that he tosses a coin and so the probability for each is 1/2. I switch to #3 or #2 and lose. The probability of losing is 1/3 * 1/3 * 1/2 + 1/3 * 1/3 * 1/2 = 1/9. Winning 2/9. Due to symmetry, the same probabilities hold true whether the car is behind doors #2 or #3. So the probability tree shown in Exhibit B holds and, in summary, the probabilities are 2/3 for winning and 1/3 for losing if I switch.

In 1999 this was the end. But my readers did not accept this "explanation." They argued and searched for a simpler, non-math solution. They wanted a "real-explanation."

After a lot of soul searching, I realized that I did not really have an insight into the problem. I just trusted the decision tree result. I also began to doubt if most people

Case Number	Car behind door	You guess door	MC opens door	You switch to door	Result
#1	1	1	2 or 3	3 or 1	Lose
#2	1	2	3	1	Win
#3	1	3	2	1	Win
#4	2	1	3	2	Win
#5	2	2	1 or 3	3 or 1	Lose
#6	2	3	1	2	Win
#7	3	1	2	3	Win
#8	3	2	1	3	Win
#9	3	3	1 or 2	2 or 2	Lose

Table 1: Nine possible situations.

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really understood what probability means. So I started to search for some deeper answers.

First I wrote down the nine possible situations (Table 1).

I looked for structure in the problem, so I used a bold 12-point font for the winners and a regular 10-point font for the losers. There were three losers and six winners. I saw no particular reason to believe that any of the cases was more likely to occur than any other, so I made the assumption that each had a probability of 1/9. So there were six winners and three losers.

Did I really understand what is going on? I reviewed the table of the nine possible situations and found that:

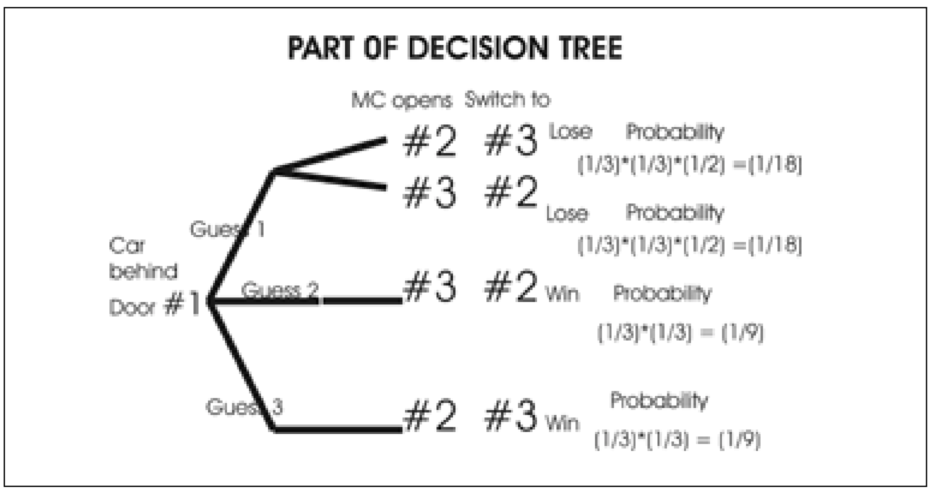
a. If I guessed right, the door opening of the emcee will make me choose the wrong door.

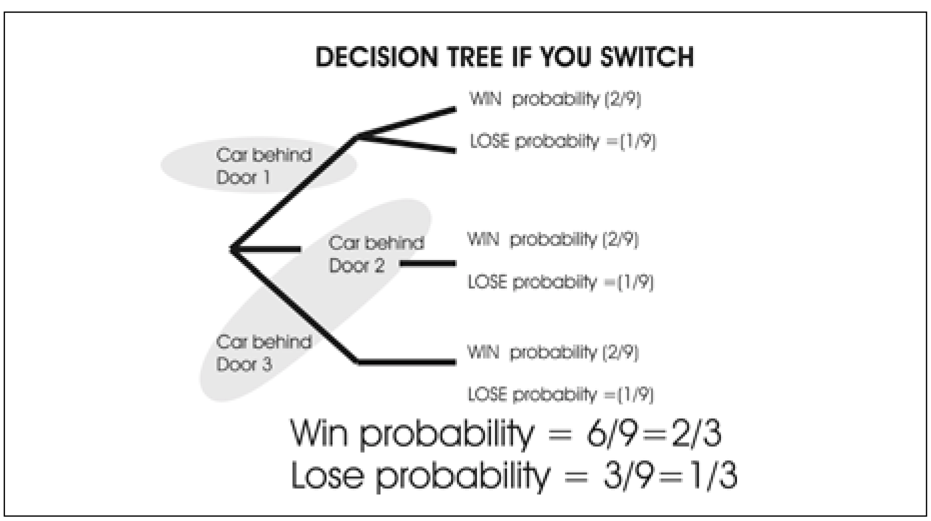
b. If I guessed wrong, the door opening of the emcee will makes me choose the right door.

It is clear that I guessed right 1/3 of the time. So switching made me right 2/3 of the time. Can I beat the dead horse more?

Look again at the "Decision Tree If You Switch" diagram and the two shaded area. The probability is 1/3 that the car is behind Door #1, in the upper shaded area. Ergo, the probability is 2/3 that it is in the second shaded area, Doors #2 or #3. Suppose the MC opens Door #3 in the second shaded area and reveals a goat. The probability is still 2/3 that the car is in the second shaded area that is in #2. So I switch.

Here's one more attempt. Suppose I take a friend with me. I will always stick to my original guess, but he will always switch. We cannot both win at the same time, but as a team we will always win. I win the car 1/3 of the time. So he wins 1 –1/3 = 2/3 of the times. QED.





What Is Understanding?

My autobiography starts with the following quotation from Samuel Johnson: "I have found you an argument; but I am not obliged to find you an understanding."

I don't think I really understood Johnson's saying when I quoted it. To explain, I have to go back to the works of my late compatriot, Michael Polanyi, the Hungarian-born physicist turned philosopher, who examined how understanding comes to scientists, and in fact to all of us, from internalizing knowledge (see his 1958 book, *Personal Knowledge: Towards a Post-Critical Philosophy*).

Personal Knowledge: How Do I Know What I Know?

Writers, actors, and artists have the ability of entering another person's skin and seeing the world through his/her eyes. Charles Dickens once said, "I was Ebenezer Scrooge when I wrote *A Christmas Carol*." Scientists understand the world in such a way through internalized knowledge. They become, in the words of Einstein, "a little piece of nature."

Richard P. Feynman said once, "Nobody really understands quantum mechanics." He gave an example of intuitive empathy toward calculus in his biographical book *Surely you must be Joking, Mr. Feynman!:*

> I often liked to play tricks on people when I was at MIT. One time, in mechanical drawing class, some joker picked up a French curve (a piece of plastic for drawing smooth curves—a curly, funny-looking thing) and said, "I wonder if the curves on this thing have some special formula?"

> I thought for a moment and said, "Sure they do. The curves are very special curves. Lemme show ya," and I picked up my French curve and began to turn it slowly. "The French curve is made so that at the lowest point on each curve, no matter how you turn it, the tangent is horizontal."

> All the guys in the class were holding their French curve up at different angles,

> holding their pencil up to it at the lowest point and laying it along, and discovering that, sure enough, the tangent is horizontal. They were all excited by this "discovery" even though they had already gone through a certain amount of calculus and had already" learned" that the derivative (tangent) of the minimum (lowest point) of any curve is zero (horizontal). They didn't put two and two together. They didn't even know that they 'knew.'
>
> I don't know what's the matter with people who don't learn by understanding: they learn by some other way—by rote, or something. Their knowledge is so fragile.

Feynman's anecdote reminds me of a problem that is sometimes given to second-year physics students. A person twirls a stone attached to a string. Suddenly the string is cut. What would the stone do? Many students believe the stone would continue flying in a circle.

But the way to understand the problem is by asking yourself, "If I were the stone, what would I do?" (Einstein, who talked about visual and muscular feelings when thinking about physics, once asked a similar question, "What would I do if I were a photon?") As long I am the stone tied to the string, I cannot escape. But once the string is cut, I am free from all constraints. Thus, according to Newton's first law, I will continue on a straight line with the same velocity. (David knew this well when he slung the stone that brought down the Philistine Goliath.)

What about the French curve and calculus? I imagine that I am the point running along the curve. I start descending, and end up ascending. Somewhere along the bottom of the valley I will run horizontally. Why? Calculus tells me so.

Back to Cadillacs

Why do people not "understand" the probability tree solution? Maybe they lack a personal feel for probability. I "understand" the problem not just because of decision trees or verbal arguments. I feel that I do have a personal, internalized knowledge of probability. I did not learn it in school or from books.

On my desk is a roulette wheel. When I spin it, I receive a visual and muscular feeling about probability or randomness. I do not see "random numbers," because numbers alone cannot be random. Sequences of numbers may be samples, finite outputs of random processes. I am the ball bouncing on my desk. Where should I stop after a dizzy spin? I have a feel for spinning coins or tossing dice.

I visualize the three doors and make a thought experiment of visualizing the problem. Pretty soon I have a gut feeling for it and I know that I should switch. Thought experiments are the solution for me, because I cannot physically try everything. Galileo thought hard about Aristotle's statement that heavy bodies fall faster than smaller bodies. What if they are tied together with a string?

Understanding the Cadillac problem comes from the proper thought experiment. Strengthening the understanding comes from decision trees, verbal explanations, and simulations. Same for other applications of probability theory. I need tools like coins, dies, wheels of fortune, roulette wheels, and others. I need a subjective, internalized feel for the problem.

The unusual thing about the Cadillac problem is that it is simple and leads to a solution "against common sense." In much the same way, the argument that the earth moves around the sun once seemed to go against common sense. But people got used to it and don't argue about it anymore.

I feel working on a "non-mathematical" solution for the Cadillac problem is not a useful effort. Quoting Euclid, "There is no royal road to geometry". ■

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Decision Sciences Institute Budget Summary FY 2003-2004

July 1, 2003-June 30, 2004

Revenues summary		
Publications	$ 54,823	
Membership Revenues	187,483	
Convention	334,830	
Total revenues		$577,136
Expenses summary		
Publications	$98,243	
Member Services	249,642	
Convention	232,030	
Total expenses		$579,915
Net Revenue Over (Under) Expenses		(2,779)
Plus Depreciation Expense (Not a cash expense)		12,661
Net Revenue Over (Under) Cash Expenses		$9,882