where for a unique solution we require C^{-1} to exist, i.e. $\det C \neq 0$. When all the securities are uncorrelated, C is diagonal and we have $f_i^* = (m_i - r)/s_{ii}$ or $f_i^* = (m_i - r)/s_i^2$, which agrees with eqn. (7.3) when $n = 1$.

Note: BRK issued a new class of common, ticker symbol BRK.B, with the old common changing its symbol to BRK.A. One share of BRK.A can be converted to 30 shares of BRK.B at any time, but not the reverse. BRK.B has lesser voting rights and no right to assign a portion of the annual quota of charitable contributions. Both we and the market consider these differences insignificant and the A has consistently traded at about 30 times the price of the B.

If the price ratio were always exactly 30 to 1 and both these securities were included in an analysis, they would each have the same covariances with other securities, so $\det C = 0$ and C^{-1} doesn't exist.

If there is an initial margin constraint of q, $0 \leq q \leq 1$, then we have the additional restriction

$$(8.2) \qquad | f_1 | + ... + | f_n | \leq 1/q$$

The n dimensional subset in (8.2) is closed and bounded.

If the rate for borrowing to finance the portfolio is $r_b = r + e_b$, $e_b \geq 0$, and the rate paid on the short sale proceeds is $r_s = r - e_s$, $e_s \geq 0$, then the m in equation (8.1) is altered. Let $x^+ = \max(x, 0)$ and $x^- = \max(0, -x)$ so $x = x^+ - x^-$ for all x. Define $f^+ = f_1^+ + ... + f_n^+$, the fraction of the portfolio held long. Let $f^- = f_1^- + ... + f_n^-$, the fraction of the portfolio held short.

Case 1. $f^+ \leq 1$

$$(8.3.1) \qquad m = r + f_1(m_1 - r) + ... + f_n(m_n - r) - e_s f^-$$

Case 2. $f^+ > 1$

$$(8.3.2) \qquad m = r + f_1(m_1 - r) + ... + f_n(m_n - r) - e_b(f^+ - 1) - e_s f^-$$

9 My Experience With The Kelly Approach

How does the Kelly-optimal approach do in practice in the securities markets? In a little-known paper (Thorp, 1971) I discussed the use of the Kelly criterion for portfolio management. Page 220 mentions that "On November 3, 1969, a private institutional investor decided to ...use the Kelly criterion to allocate its assets." This was actually a private limited partnership, specializing in convertible hedging, which I managed. A notable competitor at the time (see Institutional Investor, 1998) was future Nobel prize winner Harry Markowitz. After 20 months, our record as cited was a gain of 39.9% versus a gain for the Dow Jones Industrial Average of +4.2%. Markowitz dropped out after a couple of years, but we liked our results and persisted. What would the future bring?

It is now May, 1998, twenty eight and a half years since the investment program began. The partnership and its continuations have compounded at approximately 20% annually with a standard deviation of about 6% and approximately zero correlation with the market ("market neutral"). Ten thousand dollars would, tax exempt, now be worth 18 million dollars. To help persuade you that this may not be luck, I estimate that during this period I have made about $80 billion worth of purchases and sales ("action", in casino language) for my investors. This breaks down into something like one and a quarter million individual "bets" averaging about $65,000 each, with on average hundreds of "positions" in place at any one time. Over all, it would seem to be a moderately "long run" with a high probability that the excess performance is more than chance.

10 Conclusion

Those individuals or institutions who are long term compounders should consider the possibility of using the Kelly criterion to asymptotically maximize the expected compound growth rate of their wealth. Investors with less tolerance for intermediate term risk may prefer to use a lesser function. Long term compounders ought to avoid using a greater fraction ("overbetting"). Therefore, to the extent that future probabilities are uncertain, long term compounders should further limit their investment fraction enough to prevent a significant risk of overbetting.

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