8 A Case Study

In the summer of 1997 the XYZ Corporation (pseudonym) received a substantial amount of cash. This prompted a review of its portfolio, which is shown in Table 8.1 in the column 8/17/97. The portfolio was 54% in Biotime, ticker BTIM, a NASDAQ biotechnology company. This was due to existing and historical relationships between people in XYZ Corp. and in BTIM. XYZ's officers and directors were very knowledgeable about BTIM and felt they were especially qualified to evaluate it as an investment. They wished to retain a substantial position in BTIM.

The portfolio held Berkshire Hathaway, ticker BRK, having first purchased it in 1991.

(a) **The constraints.**

Dr. Quaife determined the Kelly optimal portfolio for XYZ Corp. subject to certain constraints. The list of allowable securities was limited to BTIM, BRK, the Vanguard 500 (S&P 500) Index Fund, and T-bills. Being short T-bills was used as a proxy for margin debt. The XYZ broker actually charges about 2% more, which has been ignored in this analysis. The simple CAPM (capital asset pricing model) suggests that the investor only need consider the market portfolio (for which the S&P 500 is being substituted here, with well known caveats) and borrowing or lending. Both Quaife and the author were convinced that BRK was and is a superior alternative and their knowledge about and long experience with BRK supported this.

XYZ Corp. was subject to margin requirements of 50% initially and 30% maintenance, meaning for a portfolio of securities purchased that initial margin debt (money lent by the broker) was limited to 50% of the value of the securities, and that whenever the value of the account net of margin debt was less than 30% of the value of the securities owned, then securities would have to be sold until the 30% figure was restored.

In addition XYZ Corp. wished to continue with a "significant" part of its' portfolio in BTIM.

(b) **The analysis and results.**

Using monthly data from 3/31/92 through 6/30/97, a total of 63 months, Quaife finds the means, covariances, etc. given in Table 8.1.

TABLE 8.1 Statistics for logs of monthly wealth relatives, 3/31/92 through 6/30/97.

		Berkshire	BioTime	SP500	T-bills
Monthly	Mean	0.0264	0.0186	0.0146	0.0035
	Standard deviation	0.0582	0.2237	0.0268	0.0008
Annual	Mean	0.3167	0.2227	0.1753	0.0426
	Standard deviation	0.2016	0.7748	0.0929	0.0028
Monthly	Covariance	0.0034	-0.0021	0.0005	1.2E-06
			0.0500	-0.0001	3.2E-05
				0.0007	5.7E-06
					6.7E-07
Monthly	Correlation	1.0000	-0.1581	0.2954	0.0257
			1.0000	-0.0237	0.1773
				1.0000	0.2610
					1.0000

Note from Table 8.1 that BRK has a higher mean and a lower standard deviation than BTIM, hence we expect it to be favored by the analysis. But note also the negative correlation with BTIM, which suggests that adding some BTIM to BRK may prove advantageous.

Using the statistics from Table 8.1, Quaife finds the following optimal portfolios, under various assumptions about borrowing.

TABLE 8.2 Optimal portfolio allocations with various assumptions about borrowing.

Security Fraction			
Security	No Borrowing	50% margin	Unrestricted borrowing
Berkshire	0.63	1.50	6.26
BioTime	0.37	.50	1.18
S&P 500	0.00	0.00	12.61
T-bills	0.00	-1.00	-19.04
Portfolio growth rate			
mean	0.36	0.62	2.10
standard deviation	0.29	0.45	2.03

As expected, BRK is important and favored over BTIM but some BTIM added to the BRK is better than none.

If unrestricted borrowing were allowed it would be foolish to choose the corresponding portfolio in Table 8.2. The various underlying assumptions

are only approximations with varying degrees of validity: Stock prices do not change continuously; portfolios can't be adjusted continuously; transactions are not costless; the borrowing rate is greater than the T-bill rate; the after tax return, if different, needs to be used; the process which generates securities returns is not stationary and our point estimates of the statistics in Table 8.1 are uncertain. We have also noted earlier that because "overbetting" is much more harmful than underbetting, "fractional Kelly" is prudent to the extent the results of the Kelly calculations reflect uncertainties.

In fact, the data used comes from part of the period 1982-1997, the greatest bull market in history. We would expect returns in the future to regress towards the mean so the means in Table 8.1 are likely to be overestimates of the near future. The data set is necessarily short, which introduces more uncertainty, because it is limited by the amount of BTIM data. As a sensitivity test, Quaife used conservative (mean, std. dev.) values for the price relatives (not their logs) for BRK of (1.15, .20), BTIM of (1.15, 1.0) and the S&P 500 from 1926-95 from Ibbotson (1996) of (1.125, .204) and the correlations from Table 8.1. The result was fractions of 1.65, 0.17, 0.18 and -1.00 respectively for BRK, BTIM, S&P 500 and T-bills. The mean growth rate was .19 and its standard deviation was 0.30.

(c) **The recommendation and the result.**

The 50% margin portfolio reallocations of Table 8.2 were recommended to XYZ Corp.'s board on 8/17/97 and could have been implemented at once. The board elected to do nothing. On 10/9/97 (in hindsight, a good sale at a good price) it sold some BTIM and left the proceeds in cash (not good). Finally on 2/9/98 after a discussion with both Quaife and the author, it purchased 10 BRK (thereby gaining almost $140,000 by 3/31/98, as it happened). Table 8.3 gives the result of the actual policy, which led to an increase of 73.5%. Table 4 shows what would have happened with the recommended policy with no rebalance and with one rebalance on 10/6/97. The gains would have been 117.6% and 199.4%, respectively. The gains over the suboptimal board policy were an additional $475,935 and $1,359,826, respectively.

The optimal policy displays three important features in this example: the use of leverage, the initial allocation of the portfolio, and possible rebalancing (reallocation) of the portfolio over time. Each of these was potentially important in determining the final result. The potential impact of continuously rebalancing to maintain maximum margin is illustrated in Thorp and Kassouf (1967), Appendix A, The Avalanche Effort.

The large loss from the suboptimal policy was much more than what

would have been expected because BRK and BTIM appreciated remarkably. In 0.62 years, BRK was up 60.4% and BTIM was up 62.9%. This tells us that – atypically – in the absence of rebalancing, the relative initial proportions of BRK and BTIM didn't matter much over the actual time period. However, rebalancing to adjust the relative proportions of BRK and BTIM was important, as the actual policy's sale of some BTIM on 10/9/97 illustrated. Also, rebalancing was important for adjusting the margin level as prices, in this instance, rose rapidly.

Table 8.2 illustrates what we might have normally expected to gain by using 50% margin, rather than no margin. We expect the difference in the medians of the portfolio distributions to be \$1,080,736 $[\exp(.62*.62) - \exp(.36*.62)] =$ \$236,316 or 21.9% which is still large.

(d)**The theory for a portfolio of securities.**

Consider first the unconstrained case with a riskless security (T-bills) with portfolio fraction f_0 and n securities with portfolio fractions $f_1, ..f_n$. Suppose the rate of return on the riskless security is r and, to simplify the discussion, that this is also the rate for borrowing, lending, and the rate paid on short sale proceeds. Let $C = (s_{ij})$ be the matrix such that $s_{ij}, i, j = 1, ..., n$, is the covariance of the ith and jth securities and $M = (m_1, m_2, ..., m_n)^T$ be the row vector such that $m_i, i = 1, ..., n$, is the drift rate of the ith security. Then the portfolio satisfies

$$f_0 + ... + f_n = 1 \tag{8.1}$$

$$\begin{aligned} m = f_0 r + f_1 m_1 + ... + f_n m_n &= r + f_1(m_1 - r) + ... + f_n(m_n - r) \\ &= r + F^T(M - R) \\ s^2 &= F^T C F \end{aligned}$$

where $F^T = (f_1, ..., f_n)$ and ^{T} means "transpose", and R is the column vector $(r, r, ..., r)^T$ of length n.

Then our previous formulas and results for one security plus a riskless security apply to $g_\infty(f_1, ..., f_n) = m - s^2/2$. This is a standard quadratic maximization problem. Using (8.1) and solving the simultaneous equations $\partial g_\infty / \partial f_i = 0$, $i = 1, ..., n$, we get

$$F^* = C^{-1}[M - R] \tag{8.2}$$

$$g_\infty(f_1^*, ..., f_n^*) = r + (F^*)^T C F^*/2$$

TABLE 8.3 Results: What XYZ Corp. actually did.

ACTUAL

	8/17/97	10/9/97	10/9/97	2/9/98	2/9/98	3/31/98
Years from 8/17/97	0.00	0.15	0.15	0.48	0.48	0.62
Security						
Berkshire	41,900.00	45,600.00	45,600.00	53,450.00	53,450.00	67,200.00
BioTime	8.75	22.75	22.75	12.88	12.88	14.25
T-bills						
Total:						
	Original Investment	Investment	New Investment	Investment	New Investment	Investment
Berkshire	209,500	228,000	228,000	267,250	801,750	1,008,000
BioTime	581,543	1,512,011	1,102,511	623,948	623,948	690,584
T-bills	289,694	291,381	700,881	710,393	175,893	176,859
Total:	1,080,736	2,031,392	2,031,392	1,601,591	1,601,591	1,875,443
Increase from 8/17/97		950,656	950,656	520,855	520,855	794,706
	Fraction	Fraction	Fraction	Fraction	Fraction	Fraction
Berkshire	0.19	0.11	0.11	0.17	0.50	0.54
Biotime	0.54	0.74	0.54	0.39	0.39	0.37
T-bills	0.27	0.14	0.35	0.44	0.11	0.09
Total:	1.00	1.00	1.00	1.00	1.00	1.00
	Shares	Shares	Shares	Shares	Shares	Shares
Berkshire	5	5	5	5	15	15
BioTime	66,462	66,462	48,462	48,462	48,462	48462

TABLE 8.4 Results: If XYZ Corp. had followed the Quaife recommendations.

	QUAIFE RECOMMENDED					
	Start		No Rebalance	One Rebalance		
	8/17/97	8/17/97	3/31/98	10/6/97	10/6/97	3/31/98
Years from 8/17/97	0.00	0.00	0.62	0.14	0.14	0.62
Security						
Berkshire	41,900.00	41,900.00	67,200.00	46,100.00	46,100.00	67,200.00
BioTime	8.75	8.75	14.25	25.33	25.33	14.25
T-bills						
Total:						
	Original Investment	Balance Investment	Investment	Investment	Rebalance Investment	Investment
Berkshire	209,500	1,621,104	2,599,957	1,783,602	3,385,420	4,934,929
BioTime	581,543	540,368	880,028	1,564,494	1,128,473	634,766
T-bills	289,694	-1,080,736	-1,128,608	-1,091,149	-2,256,947	-2,334,427
Total:	1,080,736	1,080,736	2,351,377	2,256,947	2,256,947	3,235,269
Increase from 8/17/97			1,270,641	1,176,210	1,176,210	2,154,532
Excess over suboptimal Board investment strategy (ACTUAL)			475,935			1,359,826
	Fraction	Optimal Fraction	Fraction	Fraction	Fraction	Fraction
Berkshire	0.19	1.50	1.11	0.79	1.50	1.53
Biotime	0.54	0.50	0.37	0.69	0.50	0.20
T-bills	0.27	-1.00	-0.48	-0.48	-1.00	-0.72
Total:	1.00	1.00	1.00	1.00	1.00	1.00
	Shares	Shares	Shares	Shares	Shares	Shares
Berkshire	5	39	39	39	73	73
BioTime	66,462	61,756	61,756	61,756	44,545	44,545